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Dr. *GREGORY's*

ELEMENTS

OF

CATOPTRICS

AND

DIOPTRICS.

To which is added,

I. A Method for finding the *Foci* of all *Specula* as well as *Lens's* universally. As also for *Magnifying* or *Lessening* a given Object by a given *Speculum* or *Lens* in any assign'd Proportion, *&c.*

II. A *Solution* of those *Problems* which are left undemonstrated.

III. A particular Account of *Microscopes* and *Telescopes*, from Mr. *Huygens*.

WITH

An INTRODUCTION shewing the Discoveries made by *Catoptrics* and *Dioptrics*.

By W. BROWNE, *A. M.* & Med. Pract.

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Advertisement.

THE READER *is desir'd to pardon some Mistakes which I have made in the fourth and fifth Pages of the* Introduction, *concerning the* Moons's Atmosphere. *I wrote it two Days after the Eclipse, when the Accounts that came in from several Parts seem'd to agree with what I laid down; but it appears by the Observations made upon the* Glory *which appear'd about the Moon, of which we have since had Accounts, that it was really concentric with the Sun; besides, its breadth being between a seventh and eighth of the Moons Diameter, or* $7\frac{1}{2}$ *Minutes; if we allow a Mile for every Second, the Moon's Atmosphere wou'd at that rate be sensible for the height of* 450 *Miles, whereas the Earth's Atmosphere is sensible no higher then* 45 *or* 50 *Miles from the Earth's Surface: So great a Disproportion does not seem probable, especially since we can so distinctly see the Spots of the Moon. Notwithstanding all this, we do believe that the Moon has an Atmosphere, but too thin and low to agree with the* Phænomenon *of the Glory visible when the Eclipse was total.* I had not given the Reader the trouble of this Correction, but that the first half Sheet of this *Introduction* was Printed off before I cou'd alter it.

W. BROWNE.

INTRODUCTION,

Shewing the *Discoveries* made by CATOPTRICS and DIOPTRICS.

IT is a great Encouragement for those who would take the Pains to perfect themselves in any Science, to be first inform'd of what Service it will be to understand it: And since there is no part of Learning of so real and general benefit to Mankind, as this of *Catoptrics* and *Dioptrics*, it is but Justice both to the Subject and Reader, to give some account of the many wonderful Discoveries which we owe entirely to this Science.

The *Sight* of Man is of it self confined to very narrow Views, and though it takes in a great part of the Creation at once, yet

all is represented in Miniature and imperfectly. The naked *Eye* sees only so much o external *Objects* as is sufficient to move the principal Passions, and give notice of what more immediately concerns the Safety and Happiness of the Animal. What is more than this, was left as a Subject for our Curiosity, upon which we might exercise those Faculties which are bestow'd by our bountiful Creator for this very End, of searching into the astonishing Mechanism of all his Works, and from thence enlarging our Idea of his Greatness. *Objects* plac'd at a great distance, whether upon the Surface of our Earth, or in the Heavens, are seen under so small an Angle, that their Parts are not to be distinguish'd one from another; and by this means those distant charming Scenes of Nature were hid from us, which the Study of *Catoptrics* and *Dioptricks* has since laid open to our View. This noble part of Knowledge teaches us how by a due Position of *Glasses* ground into certain Figures, we may enlarge the Diameters of the Heavenly Bodies, and all such *Objects* to which we are allow'd no nearer approach, in what proportion we please, and view them as perfectly and distinctly as if we could summon them before us, and command them to the End of our *Telescope*. This has brought us into

into a perfect Acquaintance with those surprizing parts of the Creation, which are far separate from this Globe of ours, and with which we are allow'd no Commerce but looking. We can now perceive the *Sun* to be a vast Globe of Fire, and by the different *Phases* of all the *Planets*, that He is the Fountain of all their *Light*. The Surfaces of most of them appear like so many *Maps* of Land and Water, and there are few now but allow both them and the *fix'd Stars* some nobler use than to twinkle upon us o' Nights. By fixing upon some remarkable *Spots* upon their Surfaces, and observing how they shift their Position, and in what time they again return to the same place, we determine the Motion of these Bodies round their *Axes*, and the Time in which that Revolution is perform'd. Several *secondary Planets* or *Satellits*, which were too small for the naked *Eye*, are now descern'd to move round *Jupiter* and *Saturn*, as the *Moon* round our Earth; and about the last of them is seen the particular *Phænomenon* of an *Annulus* or *Ring*. Nor is the Discovery of these *Satellits* meerly speculative, but of prodigious use and advantage; for their *Eclipses* have determin'd the *Velocity* of *Light*, as *Romer* has shewn, and are so frequent as to be the most constant appearance

the Heavens afford us at preſent, for the Solution of the great and valuable Problem of the *Longitude*. The Diſtances, Magnitudes, and Motions of all the Heavenly Bodies, and even the Irregularities of the *Moon*, have by this means been ſo nicely obſerv'd, and by the Power of Numbers reduc'd within ſome few Tables for common and eaſy Uſe, that their Places for any determinate Inſtant of Time to come are now to be predicted as eaſily, and almoſt as exactly as we could wiſh.

The nice Prediction Dr. *Halley* gave of the late *Solar Eclipſe*, total at *London*, a rare ſight in our part of the *Globe*, is ſuch an Inſtance of the great Perfection to which we are arriv'd in theſe Matters, as has amaz'd thoſe unthinking Gentlemen, who were only to be rowz'd out of their Security in Ignorance by the apprehenſions of *Doomſday*. And here, in a particular manner, let me congratulate the learned World upon the long look'd for Diſcovery of the *Lunar Atmoſphere*, which the *Teleſcopic* Obſervations upon this noble *Phænomenon* have at laſt demonſtrated: For beſides the remarkable Dimneſs of the *Sun*'s Light as the *Moon* was coming on, in compariſon with that which was obſerv'd as ſhe went off; the Reaſon of which might very well be, that the preceding part of the *Moon's At-*

moſphere

mosphere which first came over the *Sun*, having enjoy'd a *Lunar* Day of 15 of our Days, must be full of Vapours rais'd to a great Height by the constant Action of the *Solar* Heat, and so give a considerable Interruption to his Rays in their passage to us, whilst that which last left the *Sun's* Disc having enjoy'd a *Lunar* Night of the same length, must have its Vapours condensed and sunk upon the Surface of the *Moon*, and so become clear and pellucid; besides this, the *Ring of Light*, which was visible round the *Moon*, during the total Darkness, was observ'd to be concentrical with the *Moon* her self, even by those who saw the total Darkness little more than momentary, which is a plain Demonstration of a *Lunar Atmosphere*. The prodigious Distances of the *fix'd Stars* beyond that of any of our *Planets*, is, besides their little or no *Annual Parallax*, plainly deducible from the *Telescope*; for the longest that ever was made, and which perhaps represents their *apparent Diameters* one or two hundred times larger than the Truth, has been so far from magnifying them, that by cutting off those irregular Rays which hinder us from distinguishing the true Termination of their Orbs, it makes them appear something lessen'd; besides that Mr. *Huygens* has given a Method of even computing these

these Distances by means of the *Telescope*.

It is now reckon'd no absurd Notion to conceive these *fix'd Stars* as so many *Suns*, probably at as great distances one from another as they are from us, and every one their *System* of inhabited *Planets* circling round it: And perhaps the number of those which we see, counted by *Hevelius* to be 1888, may bear little or no proportion to those others that may be dispers'd thro' the vast Regions of the *Universe*, at such distances from our little *Ball*, that no assistance can ever help us to a sight of them. A Notion that gives, surely the most just and noble Sentiments that the Mind of Man can entertain of an Almighty Author! That the *milky Way* in the Heavens, which we behold in a clear Star-light Night, is nothing else but a continued Cluster of such *fix'd Stars*, is a Truth of which we are assur'd by the *Telescope*. And to the same help it is we owe all we know of those Heavenly Bodies call'd *Comets*; their Distance, Magnitude, and Motion round the *Sun* in such *Eccentrical Orbits*, as come some of them very near to right Lines. To what a surprizing height this *Cometical Astronomy* has been carried by the present Age, notwithstanding the Observations we have been able to make upon these Bodies are so few,

and thoſe made by our Predeceſſors ſo imperfect, may be ſeen in the Writings of thoſe incomparable Aſtronomers, Sir *Iſaac Newton*, Dr. *Gregory*, and Dr. *Halley*. That in every clear Morning and Evening we ſee the *Sun* for ſome time before he riſes, and after he ſets, is a *Paradox* only to be unriddled by *Dioptrics*; and if we would know the true place of any Heavenly Body elevated not many Degrees above the *Horizon*, the ſame Science tells us that here ſeeing is not believing, but that we muſt correct our *Eyeſight* by a *Table of Refractions*. It is true the *Ratio* of Refraction of the *Atmoſphere* very near the *Horizon* does not obſerve a conſtant Rule, becauſe there happens a very great variety in the Accumulation of Vapours about thoſe Parts: But then this Variation depends pretty regularly upon the Poſition of the *Sun* above or below the *Horizon*, and the different State of the Weather; and if in the Morning or Evening we ſee the lower parts of a diſtant Tower or Mountain thro' a *Teleſcope* fix'd in Poſition, we ſhall find the upper parts of the ſame Tower or Mountain in the ſame place, if our Obſervations be made nearer Noon, and juſt at Noon the ſame Object will be ſeen loweſt of all, as the accurate Mr. *Huygens* has obſerv'd; and this difference is greater in cold and moiſt than

in

in hot and dry Weather, and tho' not in a proportion always certain, yet conſtant enough for Phyſical Purpoſes. The *Crepuſculum* or *Twilight* is determin'd from the Rays of the Sun below the *Horizon*, firſt refracted at their entrance into the Earth's Atmoſphere, and then reflected from that part of it near our *Horizon*, or rather from the contiguous Surface of the *Æther*, as from a *Concave Speculum*: And the height of the *Atmoſphere* has been attempted from this Theorem by *Varenius*, but the Air being a *Medium* of different Denſity, and conſequently of different Refraction at different Diſtances from the Earth, refracts the Rays of the *Sun* into *Curves*, and makes that Solution leſs exact. In ſhort, without the aſſiſtance of *Teleſcopes*, *Aſtronomy* could have come to nothing, and our Obſervations on the Heavens had gone little further than foretelling a fine Morning from the ſetting of the *Sun*, or a Shower of Rain from the courſe of the Clouds. Theſe Inſtances are ſufficient to ſhew that all the noble Diſcoveries of the Heavens, of which the preſent Age may ſo juſtly boaſt, are deriv'd from the Knowledge of *Catoptrics* and *Dioptrics*; and whatever Improvements are hereafter to be made, can be expected from no other Fountain.

I ſhall now deſcend to a Proſpect no leſs amazing, which the ſame Science opens to

us in the minute parts of the Creation. The Difficulty which hinder'd the naked *Eye* from examining the smallest Particles and subtle Texture of those Bodies that are always under our Command, was, that when such *Objects* are brought near enough the *Eye* to have their least Parts subtend a sensible Angle, they become without the limits of *distinct Vision*. For as long as the *Pupil* of the Eye can, by the Circular Fibres of the *Uvea*, be contracted in proportion as the *Object* is brought nearer, the *Cones of Rays* from each *Point* may still be look'd upon as *Cylinders*, and will consequently be brought to a Point in the *Focus* of the *Eye*, which is at the *Retina*, and still make *distinct Vision*: But this Contraction of the *Aperture* of the *Pupil* holding no nearer than about four Inches from the *Eye*, if the *Object* is brought nearer than this, the encreas'd Magnitude is of no farther service, because the *Rays* from each *Point* must be now consider'd as *diverging*, and will consequently after refraction at the *Eye* be made to converge to a *Focus* beyond the *Retina*, or *Focus of parallel Rays*, and so make *confused Vision*; and the nearer the *Object* approaches, the farther is its *Image* projected beyond the *Focus* of the *Eye*, and becomes so much the more confused. *Dioptrics* teaches us to remedy this Inconveniency

two ways; the first is by looking thro' a *hole* prick'd in a thin *Plate*, suppose of *Lead*, whose *Aperture* must be so much the smaller as the *Object* is nearer, for this supplies the place of a farther Contraction of the *Pupil*: But because this lessening the the *Aperture*, excludes a great many *Rays* from each *Point*, and so diminishes the *brightness* of the *Image*, and that in a *duplicate Ratio* of the *Diameter* of the lessen'd *Aperture*, the same Science has also pointed out to us the more curious Invention of the *Microscope*. By means of this, we discern the admirable Range of the *constituent Particles* of all such Bodies as come within our nearer view and acquaintance. The *Cuticula* or outward Skin of the Humane Body is found to be composed of several *Strata* of *Scales*, lying one over another in different Numbers, according to its different thickness in different places: Between these *Scales* the *miliary Glands* dispersed over the Surface of the whole Body, are seen to send out their *Excretory Ducts*, thro' which we perspire; and about one of these *Scales* the *Microscope* reckons near 500 such *Ducts*, and that one Grain of Sand will cover 250 such *Scales*; so that one Grain of Sand will cover 125000 *Orifices* of these *Excretory Ducts*. A Discovery that must make us bless our selves, and

stand

ſtand aſtoniſh'd at the *Infinity* of the *Creator*, when the Creature is ſo much beyond our Comprehenſion! The inquiſitive Mr. *Lewenhoek* has oblig'd the World with a prodigious Number of ſuch ſurprizing Truths, which the curious Reader will find among his Writings. The extream Ductility and Minuteneſs of the Particles of *Gold* is no leſs wonderful; for a piece of Silver gilt with *Leaf Gold*, and drawn into the fineſt gilt Wire, whoſe Diameter is $\frac{1}{386}$ of an Inch, and the thickneſs of the Skin of *Gold*, (as Dr. *Halley* has, from the Specifick Gravities of the two Metals, computed it) not above $\frac{1}{134500}$ of an Inch, diſcovers not the leaſt Particle of Silver through the Pores of this Skin of Gold, tho' view'd by the *Microſcope*. The Particles of the Duſt which flies like Smoke out of the *Fungus Pulverulentus*, or *Puff-Ball* when burſt, are diſcern'd to be perfect Spherules of an Orange Colour, ſomething tranſparent, and their Diameters not above $\frac{1}{50}$ of that of an Hair; ſo that a Cube of an Hairs breadth would contain 125000 ſuch Spherules. The *Circulation of the Blood*, that noble Diſcovery of our Immortal *Harvey*, is now made viſible in the tranſparent parts of Animals, ſuch as the Fins and Tails of Fiſhes, and the Feet of Frogs; and the *Anaſtomoſis* of the Arteries and Veins put out of

Queſtion. It is no leſs inſtructive than curious to behold the different Organization of the leſſer *Species* of Animals, as the regular Armour of the *Flea*, the jagged Proboſcis of the *Tick*, and the Briſtles of the *Mite*; and in a *Lowſe* as he he ſtirs his Legs, you ſee the Motion of the *Muſcles* of his Body, whoſe *Tendons* ſeem all to be united in a longiſh dark Spot in the middle of his Breaſt, and the like Motion is obſervable in the *Muſcles* of the ſeveral Articulations of the Legs, and in thoſe of the Head as he ſtirs his Horns, there alſo appears a great variety of Branchings of Blood-Veſſels, and the pulſe regularly beating in ſeveral Arteries, and even the *Periſtaltick Motion* of the *Inteſtines*, continued from the Stomach all the way to the *Anus*, which is alſo to be ſeen in the *Flea*, and ſeveral ſorts of tranſparent *Maggots* and *Caterpillars*. Beſides theſe, the *Microſcope* has preſented us with an infinite variety of little *Animals*, with which the naked Eye can have no Acquaintance. They are obſervable in different Shapes and Sizes about the green Weeds growing in Water, in ſeveral Aromatick Infuſions, and in the ſtanding Water in the Hollow of the *Cabbage* and *Teazle*, but in ſuch Numbers in that which drains from an Horſe Dung-Hill, that they appear ſometimes as thick as Bees in a Swarm, or Ants on an Hillock, and muſt be diluted with fair

Water

Water to ſeparate their different *Species.* The *Animalcula* in the *Semen virile* are of all, the Subjects moſt worthy our Notice and Admiration, becauſe from this little ſhapeleſs Creature, we have reaſon to believe that the glorious Frame of Man himſelf ariſes; and this the rather, becauſe in the Seeds of Plants and Trees the *Microſcope* diſcovers the future Plant and Tree already form'd, and the *Semen Maſculinum* of other Animals, as Bucks, *&c.* are found to be furniſh'd with its *Animalcula:* Where it is to be remark'd, that ſometimes the Viſcidity of the *Semen* hinders the Succeſs of Obſervations of this kind, and muſt in ſuch Caſes be diluted with a little warm Water. This Theory of Generation is handſomely and at large explain'd in the *Philoſ. Tranſactions* by Dr. *Garden.* The uſe of *Microſcopes* has found that loathſome catching Diſtemper the *Itch*, to be occaſion'd by the Depredations made upon the Skin by a certain *Species* of voracious *Animalcula*, which are deſcrib'd in the *Phil. Tranſ.* by a Foreigner, in a Letter to Dr. *Mead*; and indeed ſeems to promiſe the finiſhing Hand to the Science of *Medicine:* For if we can once by a ſufficient Number of Experiments, determine the different change of the Texture of the Blood in every different Diſtemper from that which it enjoys in its

natural

natural and healthful State, and by mixing the ſmalleſt Particles of ſeveral Medicines with it, find out thoſe which will again reduce it to that natural State, there ſeems to be nothing more wanting to the *practic Part*; and if the true *Mechanic Theory* of all theſe different Changes be ever to be known whilſt we live in this Cloud of Fleſh, I'm ſure we muſt have the *Data* for it from the *Microſcope*. The Method of eſtimating the Magnitude of *Microſcopical Objects* ſeen by a ſingle *Lens* only, being ſo eaſy that any one who knows ever ſo little of *Plane Trigonometry* will eaſily hit of it himſelf, is not mention'd in this Book; beſides that it is already given by Dr. *Keill* in his *Phyſical Lectures*; where he ſhews that an *Animalculum* placed at the the Diſtance of $\frac{1}{10}$ of an Inch before a ſingle *Lens*, and ſeen thro' it under an Angle of one Minute in length, is nearly $\frac{3}{100\,000}$ of an Inch long, and if its Figure were Cubical, the Magnitude of it would be $\frac{27}{1\,000\,000\,000\,000\,000}$ of a Cubick Inch. From whence he concludes with a great deal of Reaſon, that what ſome Philoſophical People dream of Angels, may very well be applied to theſe *Animalcula*, that when they have a mind to be merry, ſeveral thouſand Couples of them may lead up a Country Dance upon the Point of a Needle.

I am

I am ſenſible I need ſay no more in Recommendation of the Subject: And for ſo much of the Book as is Dr. *Gregory*'s, the very Name of the Man gives it ſufficient Reputation. But I am conſcious that part which I have attempted to add, ſtands in need of ſome Name to recommend it with which the World is much better acquainted than with mine; and for that reaſon I have obtain'd the Favour of making uſe of thoſe of Mr. *Jones* and Mr. *Deſaguliers*. Men againſt whoſe Judgment in theſe Matters their Approbation of the following Papers is the only poſſible Objection; and whoſe Names can never fail of meeting with that Eſteem which they deſerve, when fix'd to any thing of their own, however they may happen to be treated for appearing in this Place to recommend what is mine.

May 2,
1715.

W. BROWNE.

PROP.

Dr. GREGORY's

PREFACE.

THESE *Elements of* Catoptrics *and* Dioptrics, *which were Eleven Years ago read publickly in Lectures, in the University of* Edinborough, *I have composed for the Use of Young Students, in such a manner, that nothing but* EUCLID'*s Geometry is required towards the understanding them. For tho' I have likewise demonstrated from higher Principles, why Spheres and Conoids observe the same Laws, both in reflecting and refracting Light; yet those who are sollicitous only about the Properties of Plane and Spherical Surfaces, may, without the*

least Inconveniency, pass over all that. These last are what we have more especially consider'd, as also such Optical *Instruments as are made by a Combination of them; that is, whose Effects arise either from a single* Lens *or* Speculum, *or from several combined together. I have, as* KEPLER *did before me, made use of some* Postulata, *that come not quite up to Geometrical Strictness, but yet are of great Service in resolving Questions in Natural Philosophy, which wou'd otherwise be extremely intricate. If these Elements be found capable of instructing such as are less conversant in Optics, I shall have my End.*

THE

THE ELEMENTS OF CATOPTRICS AND DIOPTRICS.

INTRODUCTION.

THOSE Questions concerning the Nature of Lucid Bodies, and of Light, which usually cost Philosophical Writers so much Pains and Trouble, we have, after the Example of Mathematicians, omitted. For if they, who by their Inventions, have so much improv'd this Science, had employ'd all their Time in enquiring into the absolute Nature of its Object, and

the moſt hidden Cauſes of its *Phænemena*, not contented with deducing after a Geometrical Manner from thoſe more ſimple and eaſily obſerv'd Properties of Light, others leſs obvious; Optics had fallen much ſhort of that Perfection to which they are now arriv'd. Therefore whether Light be the Action of the Lucid Body driving on thoſe Bodies that lie next it, which likewiſe drive on others next to them, and ſo on of the reſt, none of them in the mean time ſingly moving any conſiderable Space; or whether it conſiſts, which is much more likely, of Corpuſcles projected with a very great Velocity from the Lucid Body thro' the circumambient Spaces; or whether it be of a quite different Nature, and ſuch as may hereafter, or perhaps never, be perfectly diſcover'd; yet we may eaſily be allow'd to aſſume this Property of it, which is ſimple enough, and confirm'd by Experiments, That from every Lucid Point, Rays are every way propagated in an Orb, and, in a Medium that is homogeneal, are diffuſed in right Lines (ſuch being the *ſhorteſt*) after the ſame uniform Tenour.

But if thoſe Rays meet with a Medium differently affected, whoſe Parts either ſtrike them back, or diffuſe them more or leſs, than the Parts of the former Medium did,

did, they will then ſuffer an *Inflection*, by which general Name, I wou'd, with other Authors, underſtand their Reflection, as well as Refraction. For Light ſtriking upon a Surface, that abſolutely denies it Entrance, but yet hinders not its being diffuſed after the ſame manner as before, will all of it return back the eaſieſt way it can find, diffuſing it ſelf ſtill as at firſt; this is called *Reflection* of Light, and the Science which treats of the Laws it obſerves according to the different Incidence of Rays upon Bodies of different Figures, is call'd *Catoptrics*. But if the Medium, upon which the Light ſtrikes, allows indeed a Paſſage to its Rays, but then ſo as that they muſt be either more or leſs diffuſed than before, every Ray will be inflected from the right Line, in which it was before diſpoſed to proceed, and this Inflection is call'd *Refraction*; and the Science which demonſtrates the Laws and Effects of it, is call'd *Dioptrics*.

The *Radiant*, is that from every Point of which Rays are propagated.

Parallel Rays, are ſuch as are equidiſtant from one another.

Diverging Rays, are ſuch as, if produced both ways, meet on the ſide contrary

to that towards which they move.

Converging Rays, are such as, if produced, meet on the same side towards which they move.

It must be observ'd, that this *Parallelism*, *Divergency* and *Convergency*, is to be understood of Rays proceeding all from the same Point.

The *Focus*, is that Point, in which Rays proceeding from the same Point of the Radiant, being produced, do meet; whence the *Focus* of Parallel Rays is look'd upon as infinitely distant.

The *Angle of Incidence*, is that, which is made by the Incident Ray, and a Right Line perpendicular to the Inflecting Surface at the Point of Incidence.

The *Angle of Reflection*, is that, which is made by the Reflected Ray, and the same Perpendicular.

The *Angle of Refraction*, is that, which is made by the Refracted Ray, and the same Perpendicular produced.

The two following Propositions we have assumed for *Axioms*, because they agree

gree both with Geometrical Reaſoning and Experiments.

AXIOMS.

1. A Ray of Light falling perpendicularly upon an Inflecting Surface, either proceeds directly forward, or is reflected back upon it ſelf. For ſince the Direction of the Ray to the Inflecting Surface, is, of all that can be drawn from the Radiant Point, either the *leaſt*, if the Inflecting Surface be a Plane, or perhaps, where this Circumſtance is wanting, the *greateſt*, and in both Caſes a determinate, and *only one* of its kind; the Ray will ſtill perſiſt in the ſame Direction, either proceeding forward, or returning backward. For there are innumerable Right Lines inclin'd to this *only one* in any given Angle, no one of which can conſequently claim to its ſelf the Direction of the Ray with greater Juſtice than the reſt.

2. If a Plane be ſuppoſed, produced thro' the Incident Ray, and a Right Line perpendicular to the Inflecting Surface at the Point of Incidence, the Inflected Ray will likewiſe be found in the ſame Plane: or, which is the ſame thing, every Inflection is made in a Surface that is perpendicular to the Inflecting Surface. And

this Surface shall, according to ALHAZEN, be call'd the *Plane of Inflection*. For since this Plane is, either the *least* or *greatest*, of all that can be produced thro' the Radiant Point, to the inflecting Surface, and consequently an *only one*, the Proposition is demonstrated after the same manner as the former: and indeed if we more closely consider it, we shall find the former to be only a particular Case of this latter.

THE

THE ELEMENTS OF CATOPTRICS.

Prop. I. Theor. I.

If a Ray of Light be reflected from a plane Surface, the Angle of Reflection will be equal to the Angle of Incidence. [Plate 1. Fig 1.]

Tho' the Demonstration of this Theorem belongs more properly to Physicks, and a Mathematician might very well take it for granted, as sufficiently prov'd by Experiments; yet in Compliance to Custom, we shall here give a Demonstration of it, and that such an one, as shall have no Dependance upon any Party of Philosophers whatever.

Let

Let A B signify a Ray proceeding from the Radiant Point A, and falling upon the Plane reflecting Surface, G C at B. I say its reflected Ray B E will be such, that the Angle of Reflection P B E (B P being supposed perpendicular to G C) will be equal to the Angle of Incidence P B A.

From the Radiant Point A, let fall A C perpendicular to the Surface G C. All the Rays comprehended between A B and A C, and which possess, in the reflecting Plane, the Right Line B C (the common Section of the Plane of Inflection with the reflecting Plane) wou'd, if the Medium G C *d e* were passable and homogeneous with the former E B C D, be diffused in the same constant Tenour; that is, at the Distance B *e* beyond G C, they wou'd be diffused into *d e*, A B *e* and A C *d* being supposed Right Lines. And since, by Supposition, the plane reflecting Surface B C, neither encreases nor diminishes the Diffusion of those Rays (for if it were otherwise, which no Experiments yet give us any reason to suspect; 'tis plain the Angle of Reflection as well as of Refraction wou'd, for the same Reasons, become unequal to the Angle of Incidence: For the due inflecting of which Rays, CARTESIUS has contrived such Conoids as are proper, *Geom. lib.* 2.) but only strikes them back upon the same Medium in which they were before;

fore; that is, upon a Medium where the Rays are disperſed or diffuſed after the ſame manner as at firſt; it is plain, that after they are reflected from the Surface G C, the Rays will be diſpers'd after the ſame manner, as they wou'd have been, if they had never met with the Surface C G, but, that being removed, had ſtill remain'd in the ſame Medium. In which caſe, when the Diſtance is encreas'd by the Exceſs B *e*, the Rays wou'd be diſpers'd thro' the Space *e d*. Wherefore 'tis plain, that at the Diſtance B E equal to B *e*, they will be diſpers'd thro' the Right Line D E, equal to the Right Line *d e*. But the Ray A C is reflected upon it ſelf, wherefore D E is equal to *d e*, and the Figure B C D E every way ſimilar and equal to the Figure B C *d e*; and indeed is the very ſame reflected, or revolv'd half way about the fix'd Right Line B C. Therefore if from the equal Angles E B C, *e* B C, the right Angles P B C, A B C be taken away, the Angles E B P, *e* B *p* will remain equal. But A B P is equal to *e* B *p*, therefore the Angle of Incidence P B A, is equal to the Angle of Reflection P B E. *Q. E. D.*

SCHO.

SCHOLIUM.

In laying down this Law of Reflection, whereby the Angles A B P, E B P, are made equal, great Respect is had to the *Maximum* and *Minimum*. For the Sum of the Right Lines A B, B E is a *Minimum*; that is, less than the Sum of any others drawn from the same Points A and E, to any other Point of the Plane G C: and so *vice versâ*. For it is requisite that the Course which the Ray (reflected from the Plane G C) takes from A to E, shou'd be the shortest of all. Because *Nature ever acts by the most easy and expeditious Methods.*

But if the Reflection be from a curve Surface, the aforemention'd Sum of Rays (or perhaps their Difference) is sometimes a *Maximum*. For Mathematicians know how near the Relation is between a *Maximum* and a *Minimum*, between the Sum and the Difference, and how easy the Transition is from one to the other.

COROL. I.

Hence the Angles E B G, A B C (which are likewise by some call'd the Angles of Incidence and Reflection) are also equal: for they are the Complements of the former to right Angles.

COROL.

Corol. 2.

If the Reflected Ray be look'd upon as an Incident Ray, the Incident Ray will *vice versâ*, be its Reflected Ray.

Corol. 3.

If the Angles PBE, PBA, or the Angles GBE, CBA be equal, BE will be the Reflected Ray belonging to AB.

Prop. II. Probl. I.

THE Focus *of Rays falling upon a plane* Speculum *being given, to find the* Focus *of those Rays after their Reflection from the Surface of the* Speculum. [Plate 1. Fig 2.]

From the given *Focus* A, draw the Right Line AC perpendicular to the plane *Speculum* CF, which produce to *a*, that C*a* may be equal to CA; *a* will be the *Focus* required. Let AD be an incident Ray, join *a*D, and produce it.

Because in the Triangles ACD, *a*CD the Sides AC, *a*C, are equal, and the Angles ACD, *a*CD also equal, because both right, and the Side CD is common to both, therefore the Angle ADC is equal to *a*DC; but *a*DC is equal to EDF; therefore ADC,

E

E D F are equal. And consequently D E, by *Corol.* 3. *Prop.* I. is the Ray reflected from the plane *Speculum* C F, belonging to A D. The same may be shown of any other Ray proceeding from the Point A, and falling upon the *Speculum.* From whence 'tis plain, that the Rays diverging from A, after their Reflection from the plane *Speculum* C F, will diverge from the Point *a*: that is, those Rays, whose *Focus* before Reflection was A, will after their Reflection have *a* for their *Focus.* *Q. E. D.*

Scholium.

Since the Eye in any Position, as at O, will receive the Rays thus reflected, after the same manner, as if they really had proceeded from the Point *a*, is it plain the Image must appear in that Place: Because the Rays diverging from the Point *a*, affect the Eye after the same manner, as if that were the primary Radiant.

Corol. I.

From this Prop. and *Corol.* 2. *Prop.* I. it follows, that the Rays E D, O B converging towards the *Focus* *a*, will after their Reflection from the *Speculum* C F, converge towards the *Focus* A: and that such a Position of the Plane C F, may be easily

easily assign'd, as shall make the Angle O B A equal to any given Angle; which is done by taking O B F equal to half the Complement of the given Angle to two right ones.

Corol. 2.

In a plane *Speculum*, the Image of any radiant Point is seen in that Place where the Reflected Ray O B that passes thro' the Centre of the Eye, meets with the perpendicular A C, let fall from the radiant Point upon the *Speculum*. Whence, tho' every part of the *Speculum*, except B, were cover'd or taken away, the Image will nevertheless be visible: and if that be cover'd, and all the rest open, it will not be seen at all.

Corol. 3.

The Images of Objects that are inclined to the Plane of the *Speculum*, are inclin'd to the plane of the *Speculum* after the same manner as the Objects themselves; and therefore in a plane Horizontal *Speculum*, vertical Objects appear inverted.

This and what follows will be easily understood, if the Object be conceiv'd as made up of several radiant Points; and the Image of every one be attended to: For of all these together the Image of the Object consists.

Corol.

Corol. 4.

A radiant Plane and its Image made by a plane *Speculum*, are equal and similar Figures, but not similarly placed. They differ as the Right Hand from the Left, or as a Figure engraven upon a Copper Plate, does from the Impression of it taken upon Paper. For they will fit if they come together.

Corol. 5.

Because the Right Line *a* B is equal to A B, *a* O will be equal to A B and B O together; that is, the Distance of the Image from the Eye, is equal to the Incident and Reflected Ray taken both together.

Corol. 6.

Whatsoever has been said of the Image of any Object or primary Radiant, holds true also of the Image of another Image. From whence arises that Multiplication of Images, made by two or more plane *Specula* duly posited; in which it is principally to be observ'd, that the Distance of any Image from the Eye, is equal to a Ray propagated from the primary Radiant, thro' all the intermediate Reflections to the Eye.

LEMMA.

A Ray of Light is inflected by a curve Surface after the same manner, as it wou'd be by a plane Surface, touching the Curve in the Point of Incidence. *Fig.* 3.

Let P Q be any curve Surface, (for here, in the Figure, as likewise in all Cases hereafter, the common Sections of the inflecting Surface, and the touching Plane, with the Plane of Inflection, are used for the inflecting Surface, and the touching Plane themselves: because every Inflection is perform'd in the Plane of Inflection, as is demonstrated in *Axiom* 2. And for the same Reason in the Room of a solid Figure, we make use of a plane one described in the Plane of Inflection; which saves us a great many Words) upon which the Ray A B falls at B. Now since the Ray is of a Thickness not considerable, the Particle, in the Curve Surface B, upon which it falls, will be extreamly small. But the Inclination or Direction of the Curve Surface at B, is the same with the Inclination of the Plane D E touching it in that Point: Wherefore the Inflection which depends upon the Direction, is likewise the same, whether it be conceiv'd, as occasion'd by the Particle B in the Curve Surface P B Q, or in the

plane one D B E. For as to the Inflection of the Ray A B, it matters not how the rest of the Surface is bent, if the Inclination of the small Particle B, upon which it falls, does but remain the same.

It wou'd be easy to demonstrate the same thing, as the Antients did, from hence, that the Angle of Contact D B P or E B Q, is less than any rectilineal one; and that a Plane may be found so inclined to the Plane D B E, that the Difference between the Inflections that are made by them both, shall be less than any given Inflection. For from hence it will follow, that the Inflection of the Ray A B, made at the Particle B of the Curve Surface, is no way different from that which is made at the Particle B of the plane Surface D B E, touching the Curve Surface in B.

Prop. III. Prob. II.

T*O find the* Focus *of Parallel Rays falling upon a given Spherical* Speculum, *(or to find the Image of a vastly distant radiant Point) with respect to an Eye placed in the* Axis *of the* Speculum, *which is parallel to the incident Rays.*

Thro'

Thro' A the Centre of the *Speculum*, draw the right Line A B parallel to the incident Rays, meeting the *Speculum* in B. Biſect A B in C. I ſay C is the *Focus* required.

In this, as well as in all the following Propoſitions, we ſuppoſe the Point D to be extreamly near the Point B. And this is neceſſary, in order that the reflected Ray belonging to the incident one E D, may meet with the Eye, which, by Suppoſition, is placed in A B, or the ſame produced: For the reflected Rays belonging to thoſe that fall more remote from the Point B do, after their diverging from their *Focus*, paſs beyond the Pupil of the Eye; and conſequently contribute nothing towards ſeeing the Image. Beſides, of thoſe Rays that enter the Pupil of the Eye, they that fall moſt directly, or neareſt the Middle of the Pupil; that is, that are reflected from thoſe Points that are neareſt to the Point B, conduce more towards ſeeing the Image, than thoſe that enter the Eye near the Extremes of the Pupil: Becauſe thoſe that fall moſt directly, and cloſe to one another, move the Senſe more forcibly, than thoſe that fall more oblique and ſcatter'd. For which Reaſons, we need have reſpect only to thoſe that fall neareſt to the *Vertex* B, which holds good in all the Propoſitions following.

Fig. 4. Let E D be one of the parallel incident Rays. Draw A D, C D, and produce them. Because the Point D almost coincides with B, C D will be nearly equal to C B: But by Construction, C A is equal to C B; therefore C A, C D are equal; and therefore the Angle C A D, C D A are likewise equal. But the Angle C A D equal to E D A, which is the Angle of Incidence; for the right Line A D is perpendicular to the Surface of the Sphere. Whence the Angle C D A is equal to the Angle of Incidence of the Ray E D. Therefore, by *Corol.* 3. *Theor.* I. D C is the reflected Ray belonging to the incident one E D. Moreover, the Angles at the *Vertex*, E D A, *e* D O, and C D A, N D O being equal, the reflected Ray N D, belonging to the Ray *e* D parallel to B A, and falling upon the convex Surface, will, if produced backward, go to C. And what is demonstrated of any one Ray E D taken at pleasure, is true of all the rest in the same Circumstances. Wherefore such Rays as are parallel to A B, and which when reflected conduce to Vision, if they fall upon a Concave Sphere, are, after Reflection, collected in C: and from thence again diverging, make the Image to be seen in that Place, by an Eye placed in the *Axis*. The reflected Ray belonging

to the very *Axis* it ſelf does alſo, as it were, diverge from the Point C, in the Middle between B and A: for the ſame thing happens to it, as to any other Ray reflected from B D, that cuts the right Line A B in a Point equally diſtant from the Centre A, and the Point of Incidence. But Rays that fall upon the Convex Side do, after Reflection, diverge from C, and make the Image appear in the Point C, (or which is all one, in a Point whoſe Diſtance from C is leſs than any given one) to an Eye placed in A B produced. *Q. E. D.*

Corol.

Hence, and from *Corol.* 2. *Theor.* I. it follows, that Rays diverging from C, and reflected from the Concave Surface, or converging towards C, and reflected from the Convex Surface, will be parallel to the right Line A B, joining the Centre of the Sphere, and the Point C.

Scholium. *Fig.* 5.

If with the Vertex B, the *Axis* B G, and a Parameter equal to the right Line F B, a Parabola be deſcribed, it will be the *leaſt* of all that can be circumſcribed about the given Circle F B; or the Circle F B will be the *greateſt* of all that can be inſcribed

within that Parabola, by *Corol.* 1. *Prop.* XX. *lib.* 1. *Vincentii Viviani de Maximis & Minimis.* And this Circle and Parabola will, at the Point B, *have the ſame Degree of Curvity*, (to uſe KEPLER'S Words, in *Cap.* VIII. *Prop.* XX. *Paralipom. in Vitellionem)* and are there moſt intimately united. For as the Contact of Lines is equivalent to two Interſections, and is really no way different from two Interſections infinitely near one another, as Mathematicians know very well, and for the ſame Reaſon Surfaces that mutually touch one another, have the ſame Power in inflecting Rays that fall upon the Place of Contact, and in producing other Phyſical Effects, as is ſhewn in the foregoing *Lemma*: So this more intimate Union, and which is equivalent to four Interſections, and conſequently the moſt intimate that can be between the Circle and conick Section, (as the Contact between this laſt and a right Line is of all the moſt intimate) this, I ſay, will, in Phyſical Effects, that depend upon Surfaces generated by the Revolutions of theſe Lines, produce a farther Equipollency. For as Rays that are parallel to G B, are by the concave Parabolick Conoid B D, reflected exactly to its *Focus* C, which is diſtant from the Vertex B, by a fourth part of the Parameter of the generating Parabola; ſo the ſame Rays reflected

flected from the concave Sphere, are collected very nearly in the same Point: and the same Speculation holds in several other Physical Matters.

Prob. IV. Prob. III.

THE Focus *of Diverging Rays being given, whose Distance from a given concave Spherical* Speculum, *is greater than the fourth part of its Diameter; To find the* Focus *of those same Rays after their Reflection from the forementioned* Speculum, *with respect to an Eye placed in its* Axis. Fig. 6.

Thro' A the Centre of the Sphere, and the given *Focus* E, draw a right Line, meeting the Spherical Surface in the *Vertex* B; this I call *the Axis of Radiation*; and suppose the Eye to be placed in it some where or other. In this right Line take the Point C such, that B C may be to C A, as B E is to E A. I say C is the *Focus* required.

Let any Ray proceeding from E, fall upon the Concave Surface of the Sphere at the Point D, near enough the *Vertex* B, (for we have nothing to do with those that fall more remote, because after Reflection they affect not the Eye placed in E B, as has before been shown) in which Case E D will be nearly equal to E B, and may

in Physical Matters be taken for it. Draw A D, C D. After the same manner C D is equal to C B; from whence by Construction C D will be to C A, as D E to E A. And inverting the Proportion, C D is to D A D E, as C A to E A. Wherefore by *Eucl. Elem.* VI. 3. the Angles A D E, A D C are equal; but A D E is the Angle of Incidence of the Ray E D, and consequently D C (by *Corol.* 3. *Theor.* I.) is the Ray E D reflected from the concave *Speculum*. And since the Ray E D is taken at Pleasure, it is plain the *Focus* of all the Rays diverging from E, after Reflection, will be in C, with respect to an Eye placed in the *Axis* E B. *Q. E. D.*

Corol.

Hence likewise will be given the *Focus* C, of Rays *e* D converging, in the foremention'd Circumstances, towards the given *Focus* E, and reflected from a given convex Spherical *Speculum*.

Scholium. *Fig.* 7.

Rays diverging from E, are reflected from the Concave Sphere B D converging to C, for this reason; because the Circle B D described on the Centre A (by whose Rotation the Sphere is generated) has the same Degree of Curvity with an *Ellipsis*, described upon any *Foci* C and E (found

as

as in the *foregoing Prop.*) thro' B, and generating an oblong Spheroid, by which, as is commonly known, Rays diverging from one of the *Foci*, are from the concave Surface reflected, converging towards the other of them.

Draw the lesser *Semiaxis* H K, join the right Lines C K, E K, each of which are equal to the *Semi-axis* B H. From the Centre H upon C K, let fall the Perpendicular H L.

It is plain L K is half the *Latus Rectum* belonging to the *Axis* B G, because it is a Third proportional to the right Lines C K and H K. (Which is also true, if from H the Point of Intersection of the greater *Axis*, with the right Line K H bisecting the Angle comprehended by the right Lines drawn from the *Foci* to any Point of the Curve, a right Line be let fall perpendicular upon either of the foresaid right Lines.) And consequently C L is half the difference between the greater *Axis* and its *Latus Rectum*; and is also a Third proportional to the right Lines, K C, C H, that is, to the greater *Semi-axis*, and half the distance of the *Foci*. Again, (by the Construction in the *preceding Prop.*) B C is to C A, as B E to E A; from whence by changing and inverting the Proportion, B E will be to B C (E G) as A E to A C. Therefore B E + E G. B E—B C : : A

E

E + A C. A E − A C, that is, B G is to C E, as C E to A E − A C. And taking the halves of them, B H is to H C, as H C to H A: That is, H A is a Third proportional to the greater *Semi-axis*, and half the Distance of the *Foci*; whence it is equal to half the difference between the greater *Axis* and its *Latus Rectum*. Wherefore A B is half the *latus Rectum*. And consequently the Circle B D is (*by Corol.* 1, *Prop.* XX. *lib.* 1. *Vinc. Viviani*) the greatest of all that can touch the Ellipsis B G on the inside at B. Wherefore (as is shown in the *Scholium* of the *preceding Prop.*) it will in B have the same Curvity with the forementioned Ellipsis. From whence it comes to pass, that a Sphere thence generated, performs the same Thing in reflecting very nearly, which a Spheroid generated by the Ellipsis does exactly.

PROP. V. PROB. IV.

THE Focus *of diverging Rays being given, whose Distance from a given Concave* Speculum, *is less than the Fourth Part of the Diameter of the* Speculum; *To find their* Focus *after Reflection from the forementioned* Speculum, *with respect to an Eye placed in the* Axis. *Fig.* 8. Thro'

Thro' A, the Centre of the Sphere, and the given *Focus* E, draw a right Line, meeting the Spherical Surface in B. In this take the Point C ſuch, that B C may be to C A, as B E to E A. I ſay C is the Focus required.

Let E D be any one of the incident Rays, draw A D, C D, and produce them; draw likewiſe the right Line, E R, parallel to the right Line C D.

Since the Arch B D is extremely ſmall, E D, E B, and C D, C B will be equal. Wherefore E D is to E A, as C D to C A; that is, (becauſe D C, E R are parallel) as E R to E A; therefore the right Lines, E R, E D, and conſequently the Angles, E R D, E D R are equal. But E D R is the Angle of Incidence of the Ray E D, and E R D is equal to its alternate N D A, wherefore (by *Corol.* 3. *Theor.* 1.) D N is the reflected Ray belonging to the incident one E D. And ſince E D is taken at pleaſure, it is plain that all the Rays proceeding from E, after they are reflected from the Concave Spherical Surface, will, if they be produced backwards, meet in C, or will have their *Focus* in C. *Q. E. D.*

Corol.

Corol.

Hence, and from *Corol.* 2. *Theor.* 1. may be found the *Focus* E, of the Rays N D, converging towards a given *Focus* C, and reflected from a given Concave ſpherical Surface B D.

Prop. VI. Prob. V.

THE Focus *of Diverging Rays being given; To find their* Focus *after Reflection from a given Convex ſpherical* Speculum, *with reſpect to an Eye placed in the* Axis. *Fig.* 9.

Thro' A, the Center of the Sphere, and the given *Focus* E, draw a right Line meeting the ſpherical Surface in B: In this take the Point C in ſuch manner, that A C may be to C B, as A E to E B. I ſay C is the *Focus* required.

Let E D be any incident Ray proceeding from E, draw C D and produce it. To this thro' E draw E R parallel, meeting the right Line A D, produced in R. The Arch B D being evaneſcent, for the Reaſons already given, C D will be equal to C B,

C B, and E D to E B; and therefore A C is to C D, as A E to E D, but because of the equi-angular Triangles A C D, A E R, A C is to C D, as A E to E R. Therefore A E is to E D, as A E to E R. Wherefore E R is equal to E D. Therefore the Angle E R D, or its equal N D R, is equal to the Angle E D R, that is, to the Angle of Incidence of the Ray E D. From whence it follows, that D N is the reflected Ray of the incident E D. And since E D is taken any how, it is plain all the Rays diverging from the *Focus* E, and entring the Eye, whose Position is given, will, after their Reflection from the convex spherical Surface B D, diverge from the *Focus* C. *Q. E. D.*

Corol.

Hence may be found the *Focus* E, towards which the Rays N D (before Reflection towards a given *Focus* C, whose Distance from the *Speculum* is less than the Fourth Part of its Diameter,) do, after their Reflection from a given convex *Speculum*, converge.

Scholium. *Fig.* 10.

It had been easie to have explain'd and demonstrated these Two last Propositions and

and their Corollaries, in the same Words, and with different Figures, but this wou'd have bred Confusion to Beginners. However, one *Scholium* will serve for both, to show that the Circle B D has the same degree of Curvity, with the Hyperbola described upon the *Foci* C and E, thro' the Point B, by the Rotation of which Curve round its *Axis*, is generated the Surface of an Hyperbolick Conoid, performing exactly the proposed Reflection.

Draw the second Diameter K M. To the joined right Line B K, erect K L perpendicular, meeting the *Axis* G B in L, and H L will be a Third proportional to the right Lines, B H and H K; and consequently equal to half the *latus Rectum* belonging to the *Axis* B G. And since, from the Nature of the *Foci*, the Rectangle G C B is equal to the square of the right Line K H; adding to both the square of B H; the squares of the right Lines B K and H C will be equal: Wherefore B K, H C are equal. And because of the Rectangular Triangle B K L; B L, or the Sum of half the Transverse *Axis*, and half the *latus Rectum*, is a third proportional to the right Lines H B, B K; or to the right Lines H B, H C: That is, to half the Transverse *Axis*, and half the Distance of the *Foci*. Again (by the Construction in the *Two preceeding Prop.*) B C is to C A, as B E to

to E A. Wherefore B E is to E G, as A E to A C. Therefore B E -- E G. B E + E G : : A E — A C. A E + A C, that is, B G is to C E, as C E to A E + A C. And taking the halves of them, B H is to C H, as C H is to H A: That is, H A is a third proportional to half the Tranverse *Axis*, and half the distance of the *Foci*. Therefore H A is equal to B L, or to the sum of half the Tranverse *Axis*, and half the *latus Rectum*. Wherefore A B is half the *latus Rectum*, and consequently the Circle B D is (by *Corol.* 1. *Prop.* 20. *lib.* 1. *Vinc. Viviani*) the greatest of all that can touch the Hyperbola B G on the inside at B. Therefore, as has been shown already, it is equally curve in B with the Hyperbola. And hence it is, that the Surface of the Sphere, generated by the Circle, performs *very nearly* the same Thing in reflecting, which the Surface of the Hyperbolick Conoid does *exactly*: That is, it changes the *Focus* of the Rays E, into C; or the *Focus* C into E, as has been shown in *Prop.* V. and VI.

PROP.

Prop. VII. Prob. VI.

THE Focus *of Rays falling upon a given spherical* Speculum *being given; To find the* Focus *of the same Rays after Reflection, with respect to an Eye (even any where out of the* Axis*) given in Position.*

Hitherto we have supposed the Eye placed in the *Axis* of Radiation, that is, in a right Line drawn through the radiant Point, and the Center of the reflecting Sphere, both because most optical Instruments are made after this manner, and because the Image seen by the Eye so placed, is much more lively and distinct than any other, because it is form'd by Rays least scatter'd, and most exactly reflected, and upon this Account challenges to it self alone the Title of an Image.

But that it may appear that the Method before used in constructing Physical Problems, and demonstrating them when constructed, is more universal, and sufficient to determine the Image, seen by the Eye, however placed: Let the reflecting Sphere be signified by its greatest Circle B D, whose Plane passes thro' the radiant Point E, and O the Center of the Eye. It is required to find the Image of the Point E made

made by the reflected Rays, with reſpect to an Eye placed at O.

It is plain in the firſt Place, that the Image will be found ſomewhere in the reflected Ray paſſing thro' O. To find that Ray, this Problem muſt be ſolved, by which having two Points E and O, and (in the ſame Plane) the reflecting Circle given; it is required to find ſuch a Point in the Circumference of the Circle, that a Ray falling upon it from either of the given Points ſhall be reflected to the other of them. And this is *Prop.* XXXIX. *Lib.* V. *Optica Alhazeni*, to which he has premiſed 7 or 8 *Lemmata*, and is now commonly called *Alhazen*'s *Problem.* The Problem is in its Nature *ſolid*, and not to be conſtructed univerſally without the Interſection of a Conick Section with the given Circle. The Conſtruction of this Problem has been publiſh'd by ſeveral eminent Geometers, *Barrow*, *Sluſius*, &c. but moſt elegantly by the moſt *noble* C. *Huygens* in the *Philoſophical Tranſactions*, No. 98. We proceed therefore to determine exactly the Point it ſelf C, where the Image is ſeen in the right Line O B, (drawn as thoſe famous Men have directed).

Produce E B, O B, till they again meet the Circle in P and R. Biſect the right Lines B R, B P in S and A, and divide S B in C ſo, that S C may be to C B, as

A E to E B. I ſay the Image by the Eye in O will be ſeen in C; and not in the meeting of the reflected Ray D N with the right Line E Q, joining the Radiant Point and the Center of the Sphere, as *Euclid* in *Theor*. 17. and 18. *Catoptrica*, and others wou'd have it.

Let the Ray E D fall upon the Point D, very near the Point B; For only thoſe Rays, that fall after this manner do, after their Reflection, enter the Pupil of the Eye placed at O. Thoſe that fall at a greater Diſtance, after Reflection, paſs beſide the Eye, and conduce nothing at all to Viſion. Wherefore in inveſtigating the Point C, where the Image is form'd, D muſt be ſuppoſed to coincide with B: In which Caſe E D = E B, C D = C B, A D = A B, and S D = S B. Moreover, becauſe O B is the reflected Ray of E B, the Angles A B Q, S B Q will be equal; and the Point D approaching near to B, and at laſt coinciding with it, the Angles A D Q, S D Q coinciding with the former A B Q and S B Q (at leaſt being very little different from them) will in that Caſe be likewiſe equal. But farther, the Point D approaching to B, the ultimate Angles, or the ſmall and evaneſcent ones, D A B, D Q B, D S B will alſo be equal: For the Circle drawn thro' Q and the coinciding Points D and B; that is, deſcri-

bed on the Diameter BQ, and consequently touching the Circle BPQR on the inside, passes thro' the middle Points of the right Lines BP, BR, howsoever drawn from the Point B: And consequently the Angles DAB, DQB, DSB, in this Case, are in the same Segment of the Circle passing thro' the Points D, B, S, Q and A. These Things being laid down, which follow from the Coincidence of the Point D with B; since, by Construction, AE is to EB, as SC to CB, AE will be to DE, as SC to CD. And since in Triangles, the Sides are as the Sines of the Angles subtended by them; the Sine of the Angle ADE, or MDA, will be to the Sine of the Angle DAB, as the Sine of the Angle CDS is to the Sine of the Angle DSB: Because therefore the Consequents are equal, the Antecedents will likewise be equal, and consequently the Angles to which they relate, namely ADM and CDS are equal. But it has been shown before, that the Angles ADQ, SDQ are equal: Therefore the Angles QDM, QDC, and those opposed to them at the vertex LDE, LDN are equal. Wherefore ND is the reflected Ray of the Incident one ED. And the same may be shewn of the other Rays, that meet with the Eye at O; namely, that their reflected Rays will diverge from the Point C.

Wherefore to the Eye receiving theſe Rays only, the Image of the radiant Point E, made by Reflection, will appear at C. *Q. E. D.*

Corol. 1.

Hence it follows, that Rays converging towards the Point C, will after their Reflection from a convex Spherical *Speculum*, converge towards E, and there form the Image, for an Eye placed in the right Line B E, any where beyond E, As likewiſe that the Image of the Radiant Point C, made by Reflection from a concave Spherical *Speculum*, will, to an Eye placed in the right Line E B produc'd, as at A, appear in E.

Corol. 2.

But if the Radiant Point E be vaſtly diſtant, then the *Ratio* of the right Line A E to E B becomes a *Ratio* of Equality, and by Conſtruction S C is equal to C B: That is, the Image of the vaſtly diſtant Radiant Point ſituated in A B produced, made by a convex or concave *Speculum*, with reſpect to an Eye placed any where in B R, or the ſame produced, will be in the middle Point of the right Line B S.

The Eye being placed in the *Axis* of Radiation, that is, the right Lines B A and B S

BS coinciding with the right Line BQ, this ſame Conſtruction ſerves, and degenerates into the Conſtructions of the four preceding Propoſitions.

But the ſpherical Surface being changed into a plane one, then this Conſtruction will be changed into that of *Prob.* I. For becauſe the right Lines AE, SC are in that Caſe infinite, but their difference remains ſtill finite, the *Ratio* of them will be a *Ratio* of Equality: Therefore EB and BC, which are proportional to them, are equal. From whence it is plain, that this one Conſtruction of this *Probl.* VI. contains all the former ones, as being more ſimple, which is frequent in Geometry.

SCHOLIUM.

The fore-mention'd Properties do therefore belong to the Circle BDR, becauſe it has the ſame Curvity, at the Point B, with a conick Section deſcribed thro' B upon the *Foci* C and E, which, as is known, reflects the Rays diverging exactly from C. The Equality of Curvity in the foreſaid Caſe, is from thence manifeſt: That the Segment of the Diameter of the Section (produced if need be) cut by the equicurve Circle, is equal to the *Latus Rectum* of that Diameter: And *vice verſa.*

Prop. VIII. Theor. II.

THE *Image of a radiant Plane Surface, made by a Spherical* Speculum, *is also a Plane Surface.* Fig. 12, 13.

Hitherto we have determin'd the Image of any radiant Point with respect to an Eye, placed, either in the same *Axis* of the *Speculum* with the radiant Point, or out of it. From whence it is easy to determine the Images of radiant Bodies, because made up of radiant Points. But to the more easy Determination of it, as likewise to many other Things belonging to Practice, the *Theorem* before us will not a little conduce: In which we make use of that Image of any Point, which is seen by an Eye placed in the same *Axis* with the radiant Point. For we speak not here of those Images, that are seen by the Eye in any other Position (which yet may be determined by the help of the preceding *Prop.*) because they are only secondary and less considerable: Especially since such an Image of a right Line has been already sufficiently consider'd by the *famous Barrow*, in *Lect. Opt.* XVI. and XVII.

Let any Spherical *Speculum* be signified by its greatest Circle BD, whose Center is

is A; and the radiant Plane by the right Line F E. From A upon F E, ſuppoſe A E drawn Perpendicular, meeting the *Speculum* in the Vertex B. Find, by ſome one of the preceding *Prop.* applicable to the Caſe, the *Focus* C of thoſe Rays after Reflection, whoſe *Focus* before Incidence was E; thro' this draw the Plane C T parallel to F E. I ſay, that the Image of the Plane F E made by the *Speculum*, will be placed in the Plane C T.

From any Point F of the radiant Plane, to A the Center of the *Speculum*, ſuppoſe the Ray F A produced, meeting the *Speculum* in D, and the Plane C T in T. The reflected Ray of the Incident one F D, is, by Conſtruction, C D; but let B H be ſuppoſed the reflected Ray of F B, meeting the Ray F D in H.

Becauſe the Angle F A E is, by Suppoſition, ſmall, the Arch D B will be alſo ſmall, and almoſt degenerate into a ſmall right Line. And the Circumference of a Circle deſcribed on the Diameter B F, will paſs thro' the Points D and E; becauſe of the right Angles F E B, F D B. Whence the Angles B F D, B E D, in the ſame Segment, are equal. And ſince the Angles oppoſed at the vertex A are equal, or the ſame, F B A, E D A will be likewiſe equal: Therefore the Angles A B H, A D C, that are by *Prop.* I. equal to theſe,

are likewiſe equal. Whence the Triangles A B H, A D C are equiangular, and conſequently ſimilar. Wherefore A B is to A H, as A D to A C; and ſince A B is equal to A D, A H will be equal to A C. But becauſe of the ſmalneſs of the Angle C A T, A T is reckon'd equal to A C, therefore A H is to be reckon'd equal to A T. But H is the Image of the Point F in the radiant Plane; whence its Poſition is in the Plane C T. The ſame may after the like manner be ſhewn of any other Point in the Plane F E: From whence the Propoſition is manifeſt.

But if the Object expoſed to the *Speculum* be vaſtly diſtant, the Images of each of the radiant Points will, by *Prop.* III. be in the middle Points of the Semidiameters of the *Speculum* tending towards them: That is, the Image of a diſtant Object, will form a Spherical Surface, concentrical with the *Speculum*. But becauſe the radiant Body, by Suppoſition, is ſeen under a very ſmall Angle, ſo ſmall a Portion of the Spherical Surface as is poſſeſs'd by its Image, ſcarce differs from the Plane Surface to which A B is Perpendicular. Wherefore the Propoſition holds in all Caſes; For the Demonſtration takes Place in any other Caſe, as well as in thoſe two expreſs'd in the Figures.

COROL.

Corol.

If the Angle E A F be too great, the right Line A H will be ſenſibly leſs than A T; whence the Image of an Object ſeen under too large an Angle, from the Center of the *Speculum*, made by a concave *Speculum*, will be ſenſibly concave: and that which is made by a convex *Speculum* will appear convex.

Scholium.

Concerning the Images of radiant Surfaces, made by Spherical Surfaces, it will not be improper to take notice of ſome few things. 1. If the Object expos'd to the concave *Speculum* be diſtant from it more than by its Semidiameter, the Image of it C T, will be diſtant from the *Speculum*, more than by a fourth part of its Diameter, but leſs than by its Semidiameter; that is, it will appear hanging in the Air between the Object and *Speculum*, and likewiſe inverted; that is, the upper Parts will appear undermoſt, and the right on the left. If the Object recedes from the *Speculum*, the Image will approach to the *Speculum*; but if the Object approaches to the *Speculum*, the Image will recede from it, till meeting at laſt with the Object, it will coincide with it in the

the Center. After the ſame manner the Image of the Radiant C T, will be F E, whoſe Properties are evident by what has been juſt now ſaid. But if the Diſtance B E be ſo far encreas'd, that the Object be vaſtly diſtant, the Diſtance of the Image C T from the *Speculum*, will be equal to a fourth part of its Diameter. But if the Sun be the radiant Body; in the room of its Image will be excited a Burning (if the *Speculum* be conſiderably larger than the Image of the Sun) becauſe of the Sun's Rays being cloſer compacted in that Place. But if a Lucid Body be placed in the Middle, betwixt the *Vertex* and Center of the *Speculum*, its Image made by the *Speculum* at a vaſt Diſtance, will enlighten Objects vaſtly diſtant.

2. If C B the Diſtance of the Radiant C T, from the concave *Speculum* B, be leſs than a fourth part of its Diameter, its Image will be E F (by *Prop.* IV. and VIII.) plac'd beyond the *Speculum*, and erect as in a plane *Speculum*, and which approaches to the *Speculum*; as the Object approaches to it, and ſo on the contrary. After the ſame manner the Image of the Radiant F E, made by a Convex *Speculum*, will be C T, whoſe Properties, from what has been ſaid, are eaſily detected: For as the Radiant approaches to the *Speculum*, the Image likewiſe approaches to it; but when

when that recedes to a Diſtance even infinite, the Image will ſtop at the middle Point, between the *Speculum* and its Center.

3. Suppoſe the Rays of each Point of the Radiant were ſo inflected, as to be about forming the Image C T; but are hinder'd from it by the Intervention of the convex *Speculum* B D. From what has been ſaid above it is plain, that theſe Rays after Reflection from the *Speculum*, will make the Image F E, whoſe Properties, Situation, and Figure, with reſpect to the Object C T (if it may be call'd an Object) are eaſily found by the foregoing Rules.

In the Preceding, and all other Caſes, the *Ratio* of the Image to the Object is given, becauſe they appear under the ſame, or equal Angles from the Center of the *Speculum*. But this will be more plain by the following *Theorem*.

Prop. IX. Theor. III.

THE Radiant and its Image made by the Speculum, *are ſeen from the* Vertex *of the* Speculum *under equal Angles.*

Fig. 14, 15.

From A the Centre of the *Speculum*, upon the Radiant H F, let fall the Perpen-

pendicular A E (which, by *Prop.* VIII. will be likewise perpendicular to the Image *f h*) meeting the *Speculum* in the *Vertex* B. Join the right Lines B F, B H, B *f*, B *h*. I say the Angles H B F, *h* B *f*, are equal. Join F *f*; this will pass thro' the Center A, because by Supposition the Image of any Point is plac'd in the *Axis* of Radiation. After the same manner the Points H A, and *h* are plac'd in the same right Line. From the Nature of the Image, B E is to E A, as B C to C A: therefore B E is to B C, as E A to C A. But because of the equiangular Triangles A E F, A C *f*; E A is to C A, as E F to C *f*. Wherefore B E is to E F, as B C to C *f*. And since the Angles B E F, B C *f* are right, the Angles E B F, C B *f* will be equal. After the same manner E B H, C B *h* are equal: Whence H B F, *h* B *f* are equal. *Q. E. D.*

Corol.

The radiant Line F E H, and its Image *f* C *h*, are to one another as their Distances from the *Vertex*, B E, B C. And if the Radiant be a Surface, it will be to its Image, in a duplicate Proportion of those same Distances. From whence, if the Distance of the Radiant, and its Image from the *Speculum* be given, the Proportion

tion of the Image to the Radiant is alſo given. In like manner, if the Image of the Radiant C made by ſeveral *Specula* be given, we ſhall have the Proportion of the primary Radiant to the laſt Image. It follows farther, that the Line F H, and its Image *f h*, will be cut in the ſame Proportion by the right Line B E, joining the *Vertex* and Center: or that F E is to E H, as *f* C to C *h*.

Prop. X. Prob. VII.

To find ſuch a Poſition of the radiant Body, that its Image made by the Speculum *may be equal to any given Figure ſimilar to the Radiant: Or, which comes to the ſame thing, that the Radiant and its Image made by the* Speculum, *may be in a given* Ratio. Fig. 16.

Let the *Speculum* B D be given, whoſe Semidiameter is B A, and let the given *Ratio* be B A to A M. If the Radiant be a Line, biſect B M in E; and by ſome of the preceding Propoſitions, find the *Focus* C, correſponding to the *Focus* E. I ſay C is the Place of the radiant Line. Becauſe from the Relation of the Points C and E,

B C

B C is to C A, as B E to E A; B C will be to B C + C A, as B E to B E + E A: that is, becaufe B E is equal to E M, B C will be to B A, as B E to A M: and inverting the Proportion, B C is to B E, as B A to A M. But (by *Corol. Prop.* IX.) B C is to B E, as the radiant Line placed in C, to which B C is perpendicular, is to its Image in E: therefore the radiant Line in C is to its Image made by the given *Speculum* B D at E, as B A to A M; that is, in the given *Ratio*. *Q. E. D.*

But if the Radiant be a Surface, in the room of A M in the preceding Conftruction, you muft take a right Line, to which B A is in a *Ratio* fubduplicate of that, which it bears to A M.

THE

THE ELEMENTS OF DIOPTRICS.

CATOPTRICS being finished, we proceed to Dioptrics, by the Assistance of which we are still furnish'd with more Instruments, and such as are fitter for Use, and admitted to a nearer View of the Secrets of Nature. For Glass is easier brought to a due Figure than Metal, and preserves it longer: Nor does a *Lens* suffer so much Loss in its Polish from any Injuries of the Air, as a *Speculum* does. And it is difficult to make a *Speculum*, that shall reflect such strong and close Rays as a Glass *Lens* transmits.

But besides these Physical Difficulties, the Causes of which are not to be sought from Dioptrics, a *Lens* is preferable in

Practice

Practice to a *Speculum* for the following Reason: because a Fault in a *Speculum* produces an Error in the reflected Ray six times greater than an equal Fault in a *Lens*, when the Ray passes out of Air into Glass; and four times greater when the Ray passes out of Glass into Air. For by *Theor.* I. the Error of Incidence produces an Error twice as great in the reflected Ray, but near thrice less in a refracted Ray out of Air into Glass; and twice less in a refracted Ray out of Glass into Air, as will appear at *Schol.* 2. *Prop.* XI.

In Dioptrics, as before in Catoptrics, we suppose the Eye placed in the *Axis* of Radiation: because this Position, where only *Lenses* are used, is easiest obtain'd; but where *Specula* and *Lenses* are mixed together, that *Axis* (by *Corol.* 1. *Prob.* I.) may be inclin'd in any given Angle.

Prop. XI. Theor. IV.

A *Ray of Light, at a plane Surface of a* Medium *of different Density, is so refracted, that the* Media *remaining the same, the* Sinus rectus *of the Angle of Incidence will, in all Inclinations, bear the same Proportion to the* Sinus rectus *of the refracted Angle.* Fig. 17.

It

It is ſufficient for a Mathematician to have proved this *Theorem* by Experiment, ſo that from this laid down, he may demonſtrate the Properties of a given Figure in refracting, or inveſtigate a Figure whoſe Laws of refracting are given: Yet, for the ſame Reaſons that induced us to demonſtrate a like Catoptrical *Theorem*, we ſhall give the Demonſtration of this.

For this Purpoſe, different Perſons have uſed different and quite contrary *Poſtulata*. *Cartes* taking up the ſame *Theorem* (tho' in other Caſes, he affirms the Propagation of Light to be inſtantaneous) wou'd have it in his *Dioptricks*, that a Ray of Light is carried with a greater Celerity thro' a denſer *Medium*, as Water or Glaſs, than thro' one leſs denſe, as Air. This appear'd too groſs to *Barrow*, *Fermat*, and others, who went into a contrary and more probable Opinion. Moreover, *Barrow*, and *Maignanus* believ'd, that ſome certain thickneſs of the Ray of Light, however ſmall (which *Cartes* neglected as inconſiderable) was neceſſary to demonſtrate this *Theorem*. We have followed the Method of Geometers, and compoſed a Demonſtration far enough from any Sect, and depending only upon a very ſimple Property of Light, which we aſſumed in the beginning.

Let the Light flowing from the Point A, and comprehended in the Plane of Inflection between AB and AD, fall upon the right Line BD, the common Section of the Plane of Inflection with the plane Surface of the *Medium* BCED of different density. (We may justly consider this by it self, because the Rays which compose it, and their refracted Rays, are distinctly from the other circumfused Light, contain'd within the fore-mention'd Plane by *Ax.* 2.) Let one of the extreme Rays AD be Perpendicular to the right Line BD, whose refracted Ray DE (whatsoever be the density of the *Medium* BE) will consequently go directly on in the same right Line with AD: But let the other AB containing a certain Angle with AD, meet the right Line BD, which separates the *Media*, in B, thro' B draw KBF parallel to AE. Let the foresaid Light contain'd betwixt the right Lines AB, AD be supposed to be propagated in the *Medium* DG, betwixt the right Lines DE and BG continued *in infinitum*: That is, let BG be the refracted Ray of AB, whether the Light passes out of a rarer *Medium* into one more dense, as in *Fig.* 17. or out of a denser one into a rarer, as in *Fig.* 18. I say, the Sine of the Angle ABK, will, in any Inclination of the Ray, have the same Proportion to the Sine of the Angle GBF.

On

On the Center B and with any distance BG, describe an Arch of a Circle meeting the Ray BG in G, from which let fall G N Perpendicular to AN, intersecting the right Line KF in F.

The Light which proceeding from A is diffused into the right Line BD, will, if BG be put for the Length of the refracted Ray, be diffused within the new *Medium* DG into the right Line GN: For the Spaces into which the forementiond Light is diffused, must be computed by these right Lines parallel to the right Line BD, because at BD it loses its former degree of Diffusion, and after that enjoys a new one. The right Line B D which divides the *Media* remaining the same, suppose the *Medium* D G, in which A B is refracted into BG, were taken away, and another *Medium* DC of a different Density substituted in its Place, in which the Ray AB is refracted into BC, *Ex. gr.* meeting the Circle in C, thro' which draw the right Line C E parallel to B D, meeting A N in E, and KF in M. The right Line CE will be the space into which the Light is diffused within the *Medium* D C, the length of the refracted Ray BC continuing still the same. If from the right Lines GN, C E into which the Light is diffused, you take away the right Lines F N, ME which depend not at all upon the

Facilities of the *Media*, that is, their *readineſs in diffuſing Light*, (For the length and Poſition of the Ray A B remaining the ſame, the foremention'd right Lines F N, M E remain the ſame and invariable, howſoever the *Facilities* of the *Media* D G, D C are augmented, diminiſhed, or even annihilated.) There will remain G F, C M for the genuine Effects of the *Media* D G, D C reſpectively. Therefore ſince all Things that reſpect the Refraction of the Ray A B from the *Medium* D K, to *Media* of different Denſities D G and D C, excepting only the Denſities of thoſe *Media*, are made the ſame, (namely the *Medium* D K out of which the Light paſſes the ſame, the Angle of Incidence A B K of the Ray A B the ſame, and the length of the refracted Ray within the ſecond *Medium* D G, or D C the ſame) the forementioned right Lines G F, C M will have the ſame Proportion with the *Facilities* of the *Media* D G, D C which produced them: For all Effects are proportional to their Cauſes.

Now if the *Medium* D C, in which the Ray A B is refracted into B C, be the ſame with the *Medium* D K, in which Caſe the refracted Ray B C (if it may be call'd ſo) of the Ray A B will be in a direct Line with the Incident one A B; G F will be yet to C M as the Facility of the *Medium*

D G

D G to the Facility of the *Medium* D C, that is, by Conſtruction, to the Facility of the *Medium* DK. Moreover C M is the Sine of the Angle CBM, or the Angle A B K, which is the Angle of Incidence of the Ray A B, and GF is, to the ſame *Radius*, the Sine of the Angle G B F, that is, of the refracted Angle of the ſame Ray: Wherefore the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, whatſoever be the Inclination of the Ray A B to the refracting Plane, (for the Angle D A B to which A B K is equal, is taken at Pleaſure) as the Facility of the *Medium* D K, to the Facility of the *Medium* DG, inflecting the Ray A B into B G. But the *Media* being ſuppoſed the ſame, the Denſities of the *Media*, and their *Facilities* ariſing from thence, and conſequently the *Ratio* of theſe will remain the ſame: Therefore in every Inclination of the Ray, the *Ratio* of the Sine of the Angle of Incidence, to the Sine of the Angle of Refraction, remains the ſame. *Q. E. D.*

This *Theorem* is thus ſhortly demonſtrated in the Geometrick Phraſe. The Poſition of the Incident Ray remaining the ſame, the refracted Ray remains the ſame, whatſoever be the Length of the Incident Ray AB: Therefore no reſpect is to be had to its Length, or the right Line A B is to be taken for nothing. Therefore the

right Line F N, which bears a certain Proportion to A B, vanishes at the same Time. Whence the right Line G N, into which the Light is diffused at a given distance B G within the *Medium* D G, is changed into the only and determinate right Line G F, which is consequently appointed by Nature, for the Measure of the Facility of the *Medium* D G to which it owes its rise. *Q. E. D.*

But if in the passage of a Ray from a denser *Medium* into a rarer *(Fig.* 18.*)* its Inclination be such, that G F which is to C M in the Proportion of the *Facilities* of the given *Media,* shou'd exceed the *Semi-diameter* B G; then the refracted Ray of A B will be nothing (after the manner of an impossible Case in a *Geometrical Problem.)* But that which shou'd have been the refracted Ray, will not enter the rarer *Medium* D G, but will be reflected from its Surface, according to the Law of *Theor.* I. in which Case the Law of this present *Theorem* (as being more universal) does nevertheless take place.

The converse of this Proposition is sufficiently manifest. Namely, That the refracted Ray of any Incident is truly assign'd, when the Angle of Incidence is to the Angle comprehended by the assign'd right Line, and another drawn Perpendicular to the refracting Surface, in the Proportion

portion of the *Facilities* of the given *Media*, which is the Meaſure of the Refraction between them: For the refracted Ray of the Incident propoſed, can be no other than that aſſigned.

The Invention of the preceding *Theorem*, which is the principal one in *Dioptrics*, is commonly attributed to *Carteſius*, tho' it was known to *Willebrord Snellius*, who was dead Ten Years before *Cartes*'s *Dioptrics* was publiſh'd: For *Voſſius* at *pag*. 36. of his Treatiſe, *De Natura & Proprietate Lucis*, publiſhed at *Amſterdam*, in the Year MDCLXII, tells us, That it appear'd from *Snellius*'s Papers which himſelf had ſeen, that he had found out, that the Proportion between the Secants of the Angles, which are the Complements of the Angle of Incidence, and the Angle of Refraction to right Ones, is conſtantly the ſame: But it is known that the Secants of Angles are reciprocally, as the Sines of their Complements to right Ones; Becauſe the *Semi-diameter* is a mean Geometric Proportional between them.

Since we have happen'd to ſpeak of Secants, it is worth the taking Notice how near the inquiſitive KEPLER was towards finding out this *Theorem*, who at *Prop*. V. and VI. *Paralipom. in Vitellionem*, lays down theſe Secants for the reſpective Meaſure of Refractions.

Corol. 1.

If B G be the refracted Ray of the Incident A B, every Thing else remaining the same, B A will be the refracted Ray of the Incident G B: For the *Facilities* of the *Media*, by which the Sines of the Angles are govern'd, remain the same.

Corol. 2.

From the Demonstration of this *Theorem* it follows, that if the *Ratio* of A to B, measure the Refraction between the *Media* A and B, and the *Ratio* of A to C the Refraction between the *Media* A and C; the *Ratio* of B to C will be the measure of the Refraction between the *Media* B and C. Which is also manifest, by supposing the thickness of the common *Medium* A, between the parallel Planes of the other Two to vanish: For by this means, the Angles remaining the same, that part of the Ray, propagated thro' the Three *Media*, which is in the *Medium* A, vanishes.

Corol. 3.

Hence it likewise follows, that if the Position of the Ray A B remaining the same, the

the Poſition of the inflecting right Line B D be changed by any Angle, ſuppoſe E, the Poſition of the refracted Ray B G, will alſo be chang'd by an Angle *e*, which is to E, as the difference of I and R to I; (the *Ratio* of the Quantities I and R being ſuppoſed the ſame with the *Ratio* of the Sine of the Angle of Incidence, to the Sine of the Angle of Refraction, and the Angles ſo ſmall, that they may be look'd upon to have the ſame Proportion with their Sines) and the Angles *e* and E are ſituated on the ſame Sides of the right Lines B G, and B D reſpectively when I exceeds R, but on contrary Sides when R exceeds I. After the ſame manner, from this Propoſition may we judge of the change the Angle G B F will undergo, when the Angle A B K is changed by any other Cauſe whatever.

Scholium I.

The foreſaid Law of Refraction is confirm'd from the Wiſdom of Nature, always acting by the moſt eaſy and expeditious Methods; which we before found in *Catoptrics*, to be a legitimate *Axiom*. Namely, to the Light proceeding from A to G, ſuch a Point of Incidence B is aſſign'd, that it may perform its Journey A B G in the *leaſt Time* poſſible. Which Point

Point of Incidence may be found by the help of any Method (*Fermat*'s for Instance) which determines a *Maximum & Minimum*, but most easily after the manner following.

PROB.

TWO *Points being given in* Media *of different Densities, and the plane Surface dividing the* Media, *whose Densities are likewise given, being given in Position: To find such a Point in the foresaid Surface, that a moveable Body proceeding from one of the given Points, thro' the Point sought to the other of them, may take up the* least Time *in its Journey.* Plate II. *Fig.* 1.

Let the given Points be A and G, and let MN be the Surface dividing the *Media* of different Densities, or rather the common Section of the plane of Inflection continued thro' A and G, with the foresaid Surface dividing the *Media*: For (by *Ax.* 2.) the Point sought will be found in that. Let the right Lines I and R express the Velocities of Light in the *Media* DK, CF respectively.

Because the Velocity of a moving Body being given, the Time is as the space run, and the Space being given, the Time is

as

as the Velocity reciprocally; if neither being given, the Time will be in a compound Proportion of the Velocity reciprocally, and the Space directly. Therefore the Time in which the right Line A B, is run, will be expreſs'd by R×A B, and the Time in which B G is run by I×B G; and conſequently the Time in which the whole Journey A B G is perform'd, which is made up of both of them, will be expreſs'd by the quantity R×A B + I×B G, which muſt be a *Minimum.*

From the Poſition of the Points A and G, and of the right Line M N being given, the right Lines A D, G C, and D C which we ſhall call a, c, and d reſpectively, are given in Magnitude. Call C B x. Whence $BD = d - x$, $AB = \sqrt{d^2 - 2dx + x^2 + a^2}$, and $BG = \sqrt{c^2 + x^2}$. Therefore $R \times \sqrt{d^2 - 2dx + x^2 + a^2} + I \times \sqrt{c^2 + x^2}$ muſt be a *Minimum.*

Now this is (as Mathematicians know very well) when the *Fluxion* of $R \times \sqrt{d^2 - 2dx + x^2 + a^2} + I \times \sqrt{c^2 - x^2} = 0$. But by the *Method* of that moſt celebrated Geometer Sir *Iſ. Newton*, *for finding the Fluxions of as many Fluents as are involv'd in any given Equation*, (to which alſo the famous *Leibnit*'s *Differential Calculus* relates) which you may ſee at *Cap.* XCV. *Vol.* II. *Operum Mathemat. Walliſii*, The *Fluxion*

xion of $R\times\sqrt{d^2-2dx+x^2+a^2}+I\times\sqrt{c^2+x^2}$ is $\frac{R\times -2d\dot{x}+2x\dot{x}}{2\sqrt{d^2-2dx+x^2+a^2}}+\frac{I\times 2x\dot{x}}{2\sqrt{c^2+x^2}}$. Therefore making this equal to nothing, we have

$$\frac{I\times x}{\sqrt{c^2+x^2}}=\frac{R\times\overline{d-x}}{\sqrt{d^2-2dx+x^2+a^2}}.$$

And ſubſtituting in the room of their Valours, the right Lines themſelves expreſs'd in the Scheme, it becomes $\frac{I\times CB}{BG}=\frac{R\times BD.}{AB}$

And if $BG=AB$, then $I\times CB=R\times BD$, or $BD.\ BC::I.\ R.$

Therefore if the right Line DC be divided in B, ſo that DB may be to BC in the given *Ratio* of I to R, the Light will perform its Journey from A to G (or backward from G to A) in the ſhorteſt or leaſt Time poſſible, by going along ABG. But DB and BC are the Sines of the Angles BAD and BGC, that is, of the Angles ABK, GBF, namely, of the Angle of Incidence, and the Angle of Refraction. Wherefore that the Time of the paſſage from A to G (or from G to A) may be the *leaſt*, the Ray muſt ſo fall, that the Sine of the Angle of Incidence, may be to the Sine of the Angle of Refraction in the *Ratio* of I to R, that is in the *Ratio* of the Velocities of Light in the foreſaid *Media*, or (as is ſhewn before in the

the Demonſtration of *Prop.* XI.) in the *Ratio* of the *Facilities* of the ſame *Media*, upon which the *Velocities* of the Body moving thro' them depend.

Since there are in the Plane dividing the *Media*, an infinite number of ſuch Points (two of which are in the right Line MN) that the Light in paſſing from A to G (or from G to A) thro' any of them ſhall take up any given Time exceeding the *leaſt*; there can be no Reaſon aſſign'd, why it ſhou'd paſs thro' one of thoſe Points rather than another: Therefore it will paſs thro' none but the only and determinate one of its kind B.

Scholium 2.

When a Ray of Light paſſes out of Air into Glaſs, we obſerve the Sine of the Angle of Incidence to be to the Sine of the Angle of Refraction, as 3 to 2 in round Numbers: And in paſſing out of Air into Water, I is to R (which Symbols we ſhall uſe for the future, to expreſs the *Ratio* of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, which is the meaſure of Refraction) as 4 to 3. And on the contrary, in the Paſſage of a Ray out of Glaſs into Air, I will be to R, as 2 to 3, by *Corol.* 1. From whence the Reaſon is plain, why a Ray in Glaſs, ſtriking upon a Surface of Air,

more

more obliquely than in an Angle of Incidence of about 42 Degrees does not enter the Air, but is reflected from its Surface: For the right Line G F which is Sesquialteral of the Sine of 42°, exceeds the *Radius*, in which Case, The Light shall be reflected as has been observ'd above.

Moreover, because in the Passage of a Ray out of Air into Glass, I is sesquialteral or $1\frac{1}{2}$ of R, and in its Passage out of Air into Water, I is $1\frac{1}{3}$ of R; in its Passage out of Water into Glass (by *Corol.* 2.) I will be $1\frac{1}{8}$ of R.

And from *Corol.* 3. it follows, that the Errour of the refracted Ray, is but subtriple of the Errour of the Surface of Glass by which it was occasion'd, and on the same Side with it, when the Ray passes out of Air into Glass, but subduple of it, and on the contrary Side, when the Ray passes out of Glass into Air.

Prop. XII. Theor. V.

LET BD *be a Surface dividing* Media *of different Densities, upon which let a Ray fall proceeding from* E, *and be refracted into* DN, *the Surface dividing the* Media *remaining the same, but the* Media

dia *being transposed, I say the refracted Ray* D C *of the Ray* M D *being placed in the same right Line with* E D, *will be in the same right Line with the former refracted Ray* N D. *Fig.* 2.

Draw the right Line A D O Perpendicular to the Surface B D in D.

Since D N is the refracted Ray of E D, (by the foregoing,) the Sine of the Angle E D A will be to the Sine of the Angle N D O in the *Ratio* of I to R, and if the *Media* be transposed, the *Ratio* of I to R will be the Measure of the Refraction out of the *Medium* B D M, into the *Medium* B D E: That is, supposing M D to be the incident Ray, its refracted Ray will be such, that the Sine of the Angle M D O shall be to the Sine of the Angle A D C, as I to R: That is, by what has been just now shewn, as the Sine of the Angle A D E to the Sine of the Angle N D O. Therefore since the Angles E D A, M D O are equal, and consequently their Sines, the Sines of the Angles A D C, N D O will be likewise equal, and consequently the Angles themselves. Wherefore N D and D C lie in the same right Line. *Q. E. D.*

Corol.

If Rays diverging from E after Refraction at the Surface B D, diverge from

from the Point C, the *Media* being transſpoſed, and the Surface dividing the *Media* remaining the ſame, Rays converging towards E, will, after their Refraction at BD, converge towards the Point C.

Therefore all that can be demonſtrated of Rays diverging, may equally be applied to converging ones. Wherefore for the future, we ſhall only ſpeak of Rays diverging, leaving thoſe converging to be determin'd by this *Theorem*.

Prop. XIII. Prob. VIII.

F*ROM the* Focus *of diverging Rays being given, to find the* Focus *of the ſame Rays after their Refraction, at a plane Surface of a* Medium *of different Denſity, with reſpect to an Eye placed in the* Axis. *Fig.* 3, 4.

Thro' E the given *Focus* of diverging Rays, draw the right Line EB Perpendicular to the plane Surface BD of a *Medium*, either denſer or rarer; in this take the Point C ſuch, that CB may be to EB, as I to R. I ſay C is the *Focus* ſought.

Let fall any Ray ED. Thro' D draw AO parallel to EB. Join CD and produce it to F.

If

If the Ray E D be near enough to the right Line E B (for we ſpeak here only of thoſe Rays that fall near the *Axis*, ſince theſe only do, after Refraction, affect the Eye placed, by Suppoſition, in the *Axis* of Radiation produced, thoſe that fall more obliquely paſſing by the pupil of the Eye, and conducing nothing at all towards diſcerning the Point E, as we have before obſerv'd in *Catoptrics*) E D will be nearly equal to E B, and C D to C B. Wherefore C D is to E D, as I to R: But C D is to E D, as the Sine of the Angle B E D, or its equal E D A, to the Sine of the Angle B C D, or its equal O D F: Therefore the Sine of the Angle E D A, is to the Sine of the Angle O D F, as I to R. But the Angle E D A is the Angle of Incidence of the Ray E D: Therefore O D F is the reſpective Angle of Refraction, that is, D F is the refracted Ray of the Incident E D. And the ſame may be ſhown of any other Ray diverging from E, whence C is the *Focus* required. *Q. E. I.*

Corol. 1.

From hence and from *Corol.* 1. *Prop.* XI. it follows, that the ſame Things being ſuppoſed as before, Rays in the *Medium* B D O converging towards C, will, after Refraction, converge towards E.

F And

And if the *Media* were transposed, it follows from hence and from *Prop*. XII. That Rays in the *Medium* BDO converging towards E, will, after Refraction, converge towards the Point C; and Parallel Rays will, after Refraction at a plane Surface of any *Medium*, still remain Parallel.

Corol. 2.

If BDA be Water and BDO Air, CB will be to EB as 3 to 4: From whence it happens, that Water appears a fourth Part less deep, than it really is. But if BDA be Glass, EB will be $1\frac{1}{2}$ of the right Line CB.

Scholium.

What is demonstrated of any Incident Ray, namely, that its refracted Ray DC produced backwards, till it meets with EB, is to the Incident ED, as I to R, is also true of the perpendicular Incident EB: For its refracted Ray BC produced backwards, tho' as to its Position it be the same with the Incident, yet as to its length it is to it in the very same *Ratio* of I to R. And because this Ray passing thro' the Center of the Eye, and the others that are nearest it, are the only ones that affect the

the Senſe, from thence it is, that with reſpect to the Eye, C is the *Focus* of the Rays diverging from E, and refracted at B D. This Conſideration extends likewiſe to what follows.

PROP. XIV. PROB. IX.

TO *find the* Focus *of parallel Rays falling upon a Spherical Surface of a* Medium *of different Denſity, after Refraction. Plate* II. *Fig.* 5, 6.

Let the Spherical Surface whoſe Center is A, be expreſs'd by the Circumference of the Circle B D, thro' A draw the *Axis* A B parallel to the Incident Rays, meeting the Circle in the Vertex B; in which take the Point C ſuch, that B C may be to A C, as I to R. I ſay C is the *Focus* required.

Let any Ray fall at D, draw the right Lines A D, C D and produce them. The Sine of the Angle B A D, or its equal E D O, is to the Sine of the Angle C D A (or its Complement to two right Ones) as C D to A C, or (the Arch D B vaniſhing for Reaſons before often mention'd) as C B to A C, that is, by Conſtruction, as I

 to

to R: But EDO is the Angle of Incidence of the Ray ED, therefore DC is its refracted Ray. After the ſame manner it may be demonſtrated, that any other of the parallel Rays will, after Refraction, paſs thro' C. Therefore C is their *Focus*. *Q. E. D.*

Corollaries.

Hence and from *Corol.* 1. *Prop.* XI. and *Corol. Prop.* XII. it follows.

1. That Rays in Air parallel to the *Axis*, after Refraction at a *Spherico-convex* Surface of Glaſs, converge to a Point whoſe diſtance from the Vertex is equal to three Semi-diameters of the Sphere.

2. That Rays in Glaſs diverging from a Point diſtant from a *Spherico-concave* Surface of Air by three Semi-diameters, do, after Refraction, become parallel to a right Line drawn thro' the radiant Point, and the Center of the Sphere.

3. That Rays in Glaſs parallel to the *Axis*, after Refraction at a convex ſpherical Surface of Air, diverge from a Point, whoſe diſtance from the Vertex is equal to the Diameter of the Sphere.

4. That Rays in Air converging towards a Point diſtant beyond a concave Glaſs by the Diameter of the Sphere, after Refraction at the concave Surface of

Glaſs,

Glaſs, become parallel to a right Line drawn thro' the Center of the Sphere, and the fore-mention'd Point.

5. That Rays in Air parallel to the *Axis*, after Refraction at a concave Surface of Glaſs, diverge from a Point at three Semi-diameters diſtance from the Vertex.

6. That Rays in Glaſs converging to a Point three Semi-diameters diſtant beyond a *Spherico-convex* Surface of Air, after Refraction, become parallel to a right Line drawn thro' the Center of the Sphere and the fore-mention'd Point.

7. That Rays in Glaſs parallel to the *Axis*, after Refraction at a *Spherico-concave* Surface of Air, converge to a Point diſtant from the Vertex, by a Diameter of the Sphere.

8. That Rays in Air diverging from a Point at a Diameters diſtance from a Sphere of Glaſs, after Refraction at a convex Surface of Glaſs, become parallel to a right Line drawn thro' the fore-mention'd Point, and the Center of the Sphere.

SCHOLIUM. *Plate* II. *Fig.* 7.

The generating Circle of the Sphere B D, has the ſame degree of Curvity with an *Ellipſis* paſſing thro' B, by whoſe Rotation a *Spheroid* being made of a denſer *Medium*, collects Rays in the ambient *Medium* pa-

rallel to the right Line A B, exactly at C. For we all know, that BG, the greater *Axis* of that *Ellipsis*, ought to be to EC the Distance of the *Foci*, as I to R. Supposing then the same Construction with that at *Schol. Prop.* IV. BG is to EC as KC to HC, that is as HC to CL, that is as KC+HC to HC+CL, or to HC +CK—LK, that is as BC to BC—half the *Latus Rectum*, belonging to the *Axis* BG, (for LK has, at *Schol. Prop.* IV. been shewn equal to half the fore-mention'd *Latus Rectum*) but by the Construction of this *Prop.* the Situation of the Point C is such, That BC is to AC, as I to R, that is, as BG to EC: Therefore BC is to AC, as BC to BC—half the *Latus Rectum*, belonging to the greater *Axis*: Wherefore AC is equal to BC—half the fore-mention'd *Latus Rectum*: And consequently AB is equal to half the *Latus Rectum*. Wherefore the Circle BD is equally curve in B with the *Ellipsis* BKG (by *Corol.* 1. *Prop.* XX. lib. 1. *Vinc. Viviani de Maximis & Minimis.*) Whence it is, that a Sphere generated by the fore-mention'd Circle, performs in refracting very nearly the same Thing, which an oblong *Spheroid* generated by the Rotation of an *Ellipsis* does exactly; namely, that Rays in a rarer ambient *Medium*, parallel to the right Line BG,

after

after Refraction at its Surface, may converge to the Point C.

After the ſame manner, by the help of *Schol. Prop.* VI. it may be ſhewn, that the Circle BD is at the Vertex equally curve with an *Hyperbola* paſſing thro' B, and generating a Conoid, which being of a rarer *Medium*, does ſo refract Rays in a denſer ambient *Medium* parallel to the right Line AB, that they ſhall afterwards diverge from the Point C: Or, which returns to the ſame Thing (by *Prop.* XII.) being of a denſer *Medium*, does ſo refract Rays in that ſame denſer *Medium* parallel to the right Line AB, at their entrance into the rarer ambient *Medium*, that they ſhall afterwards converge to the Point C.

Prop. XV. Prob. X.

THE Focus *of Rays diverging and falling upon a given Spherical Surface of different Denſity, to find the* Focus *of the ſame Rays after Refraction.* Plate II. *Fig.* 8, 9, 10, 11, 12, 13, 14, 15.

Let the Spherical Surface be expreſs'd by the Circumference of its greateſt Circle BD. Thro' the given *Focus* and the

Center of the Sphere, draw the right Line E A meeting the Circumference in B, in which take the Point C ſuch, that the *Ratio* compounded of the *Ratio* of E A to A C, and of C B to B E, may be equal to the *Ratio* of I to R. I ſay C is the *Focus* required.

We ſhall make one Demonſtration ſerve for all the eight principal Caſes of this *Prop.* expreſs'd by ſo many ſeveral Figures, the four firſt of which, ſuppoſe I greater than R, the others leſs, and which differ from one another, according as the given *Focus* is ſituated on this, or that ſide of the *Focus* of parallel Rays; or according as the refracting Surface is Convex or Concave.

Let fall any Ray at pleaſure E D proceeding from E. Join A D, D C, and thro' C draw a right Line parallel to A D, meeting the right Line E D in H.

The Point D almoſt coinciding with B, C D and C B, as likewiſe E D and E B are almoſt equal. Wherefore the *Ratio* of I to R, is equal to the *Ratio* of E A to A C, and the *Ratio* of C D to E D together; that is (becauſe of A D and C H parallel) to the *Ratio* of C D to E D, and the *Ratio* of E D to D H together, or to the *Ratio* of C D to D H: But C D is to D H as the Sine of the Angle D H C, or its Complement to two right Ones A D H

or

or EDO, is to the Sine of the Angle DCH, or of the Angle ADC, or in ſome Caſes its Complement to two right Ones. Moreover, the Angle EDO is the Angle of Incidence of the Ray ED: Therefore as I to R, ſo the Sine of the Angle of Incidence of the Ray ED, to the Sine of the Angle ADC. Wherefore (by *Prop.* XI.) DC is the refracted Ray belonging to the Incident one ED; and ſince ED is taken at Pleaſure, 'tis plain the *Focus* of all the Rays proceeding from E, will be C. *Q. E. I.*

If the *Media* propoſed were Air and Glaſs, and the Ray paſs'd out of Air, into Glaſs (as in the four firſt Figures) the *Focus* C will be very eaſily found: Namely, if the third Part of the right Line EA be to AB, as EC to BC. For trebling the Antecedents, EA is to AB (or AD) as 3 EC to BC (or DC): But EA is to AD, as the Sine of the Angle EDO to the Sine of the Angle DEA, and 3 EC is to DC as thrice the Sine of the Angle EDC (or HDC) to the Sine of the Angle DEA: Wherefore the Sine of the Angle EDO is triple of the Sine of the Angle HDC, and conſequently (in theſe very ſmall Angles which have the ſame *Ratio* with their Sines) $1\frac{1}{2}$ of the Sine of the Angle ADC. Therefore DC (or D*e* lying in the ſame right Line with it)

is

is the refracted Ray of E D passing out of Air into Glass.

When the Ray passes out of Glass into Air (as in the four last Figures) the *Focus* will be found from the same Principles, by taking the Point C such, that half the right Line E A may be to A B, as E C to B C.

Corol. 1.

From this *Prop.* and XII. and *Corol.* 1. *Prop.* XI. it will be easy, from the given *Focus* of Rays converging, and falling upon a Spherical Surface of a *Medium* of different Density; to find their *Focus* after Refraction.

Corol. 2.

Hence from the *Foci* E and C, and the Vertex B being given, we may find A the Center of the Sphere, and thence the refracting Sphere it self: By taking the Point A such, that E A may be to A C in a *Ratio* compounded of the *Ratio*'s of B E to B C and I to R. Or from the *Foci* and Center being given, may be found the Vertex. The like Problem concerning Spherical *Specula*, may (by the *Correspondent Prop. in Catoptrics*) be solved with equal ease, and more elegant

Geo-

Geometrical Constructions from thence deduced.

Corol. 3.

The Geometrical Construction of this *Prob.* X. is easily deduced from what has been premised. *Plate* II. *Fig.* 16. Every Thing else remaining as before, thro' B and A draw the right Lines BQ, AN parallel, meeting the right Line EN drawn from E as you please in F and N: Make AN to AM as I to R. Join the right Line FM, and produce it 'till it meets the *Axis* EA in C. This will be the *Focus* required.

Join CN, meeting the right Line BF in Q; to this thro' F draw parallel the right Line FH intersecting the right Line EA in H.

For EA. EB : : (EN. EF : : EC: EH : : EC—EA. EH—EB : :) AC. BH. *& permutando* EA. AC : : EB. BH. Therefore CB. BE, + EA. AC : : (CB. EB, + EB. BH : : BC: BH : : BQ. BF : : AN. AM : :) I. R. Whence C is the *Focus* required.

This same Construction will serve (only changing the order of the Points E, B, A, C, M and N) if R exceed I, or if the Rays converge, or are parallel, in which Case it will be changed into that of *Prob.* IX.

IX. Or if the Concavity of the refracting Surface look towards the *Focus* E, or if BD be plane, in which Case it changes into that of *Prob.* VIII. For when the right Line AMN (divided as above) is infinitely distant from BQ, the right Line FM will be the same in Position with a right Line joining the Point F with a Point dividing BE after the same manner as AM is divided in N, supposing BE to be homologous to AM; because the Tangents of Angles are reciprocally, as the Tangents of their Complement to a right Angle.

After the same manner, if of the four Points A, B, C, and E, any other three be given, *Ex. gr.* E, B, and C, the fourth A will be determin'd; that is, the Spherical Surface passing thro' the given Point, and changing the *Focus* E into C, will be determin'd, *& vice versa.*

Scholium.

Since there is no Spheroid or Conoid generated by the Rotation of a Conick Section round its *Axis*, and consisting of a *Medium* of different Density from the ambient *Medium*, which can exactly change a given *Focus* of Rays into another given one by Refraction at a single Surface: It follows, that this Property of performing the Thing proposed pretty nearly, does therefore belong

long to a Spherical Surface; becauſe the greateſt Circle of that Sphere has the ſame degree of Curvity with *Cartes*'s *Curve*, (*Lib.* II. *Geometriæ*) by whoſe Rotation are made Surfaces, which ſeparating *Media* of different Denſity, anſwer the Problem exactly. But if the Condition of that equicurve Circle, and of the given *Foci* lying on the ſame ſide of the Center be ſuch, that the *Semi-diameter* of the Circle be a middle proportional between the Diſtances of the *Foci* from the Center of the Circle. And one of theſe *Diſtances* be to the *Semi-diameter* in a *Ratio*, meaſuring the Refraction between the given *Media*, that Curve of *Cartes*'s is chang'd into the Circumference of a Circle. In this Situation of the *Focus* a Sphere of given Denſity, generated by the Rotation of a Circle, will ſo refract Rays proceeding from one of the *Foci*, that they ſhall all afterwards diverge preciſely from the other of them.

PROP. XVI. PROB. XI.

O*NE* Focus *of a given* Lens, *being given, to find the other.*

A *Lens* is a Tranſparent Body, of a different

ferent Denſity from the ambient *Medium*, and terminated by two Surfaces either Spherical, or Plane and Spherical. A right Line Perpendicular to both its Surfaces is call'd the *Axis* of the *Lens*. The Points where the *Axis* interſects each Surface are call'd the *Vertices*, either the Vertex of *Incidence*, or the Vertex of *Emerſion*, according as it lies in that Surface upon which the Rays firſt fall, or out of which they again emerge. The *Thickneſs* is the diſtance between the *Vertices*.

The Terms being thus explain'd, the *Focus* required may, by means of a Calculation (which in Practice is to be preferr'd to the niceſt Conſtructions) grounded upon the foregoing Propoſitions, be eaſily determin'd, and Canons (which becauſe of their Difficulty to be remember'd are here omitted) eſtabliſh'd by making the Calculation general. As if from the *Focus* before Incidence being given, it were required to find the *Focus* after Emerſion: Firſt, the *Focus* of the Rays after their Refraction at that Surface of the *Lens*, upon which they firſt fall muſt be found; and this is done by *Prop.* XIII. if the Surface of the *Lens* be plane; but if it be Spherical, and the Rays parallel by *Prop.* XIV. and by *Prop.* XV. if they be diverging or converging. And having thus got the *Focus* of the Rays after Refraction at this

this firſt Surface, that is, while they are paſſing thro' the *Lens*, which is likewiſe call'd the *Focus of Tranſition*, after the ſame manner their *Focus* after Refraction at the ſecond Surface of the *Lens*, or rather at the Surface of the ambient *Medium* contiguous to this ſecond Surface, will be found; that is, their *Focus* after Emerſion from the *Lens*. *Q. E. I.*

If there were more than one *Lens*, we muſt proceed after the ſame manner with every one of them.

By the like Method, from the *Focus* made by the help of one or more given *Lens*'s being given, the *Focus* before Incidence is found, or from the optical Machine being given, the diſtance of the Object is determin'd.

Corol.

If the Geometrical Conſtruction of this Problem be deſired, it is eaſily deduced from *Corol.* 3. of the foregoing *Prop.* by aſſuming the Conſtruction there given for one Surface of a *Lens*, and repeating it for the other. *Fig.*17. For the right Lines EN, BF, AN and FM being drawn according to the Directions of that *Corol.* 'Tis plain the right Line FM will tend towards the *Focus* of Tranſition of thoſe Rays, whoſe *Focus* before Incidence was E: If therefore this

this meets with the parallel right Lines, drawn thro' *a* the Center of the hindmost Surface, and *b* the Vertex of Emersion, in *n* and *f*, and *a n* be taken to *am*, as I to R at the Egress out of the *Lens*, (that is, as R to I at the Ingress into the *Lens*) the joined right Line *mf* will meet the *Axis* B *b* in *e*, the *Focus* required after Refraction at both the Surfaces of the *Lens*. For the right Line *mf* is in the same Condition, with respect to the Emersion of the Rays from the *Lens*, in which MF was with respect to their Immersion.

In like manner, if of the Six Points A, B, E, *a*, *b*, *e*, any other five be given, the sixth may be determin'd: For Example, from the two *Foci*, the Thickness of the *Lens*, and one Surface being given, the other Surface may be discover'd.

This is the Construction, which *Barrow* receiv'd from a Friend, and placed without any Demonstration at the End of *Lect.* XIV.

Prop. XVII. Theor. VI.

If a plane Radiant Surface sends out Rays upon any plane or spherical refracting Surface, the Rays proceeding from every

every Point of the Radiant Surface will, after Refraction, have their respective Foci *very nearly in one and the same Plane, parallel to the Radiant Plane.* Plate II. *Fig.* 18.

Let any refracting Surface be expreſs'd by B D, whoſe Center is A, and a plane Radiant Surface by the right Line E F. From A upon E F, let fall the Perpendicular A E meeting B D in B. Find C the *Focus* of thoſe Rays after Refraction, whoſe *Focus* before Incidence was E, thro' which draw the Plane C T parallel to the Plane F E. I ſay, the *Foci* of Rays proceeding from every Point of the Plane E F (or the principal Image of the Plane E F, which is made up of the Images of every one of thoſe Points, with reſpect to an Eye placed in the *Axis* of Radiation: For we take no Notice here of the ſecondary Image, ſeen by an Eye in any given Situation, which *Barrow* has conſider'd in his three laſt *Lect.*) will all be poſited very nearly in the Plane C T. From the Point F taken at Pleaſure in the radiant Plane, to the Center A draw the right Line F A, meeting the refracting Surface at D, and the Plane C T in T. D C will be the refracted Ray of the Incident E D; and ſuppoſe the refracted Ray of the Incident F B to be B H, meeting the right

Line DT in H. Because the Angle EAF is supposed but small, the Arch BD is to be look'd upon as a right Line, and a Circle described on the Diameter BF will pass thro' the Points D and E, because of the Angles BEF, BDF being right. Whence the Angles EBF, EDF (namely the Angles of Incidence of the Rays FB and ED) contain'd in the same Segment, are equal, and consequently the Angles of Refraction ABH, ADC will also be equal. Therefore by reason of the Angles at A equal, the Triangles BAH, DAC are equiangular, and BA is to AH, as DA to AC, and since BA is equal to DA, AH will be likewise equal to AC: But because the Angle EAF or TAC, is very small, AT is very nearly equal to AC, and therefore AH, AT may be look'd upon as equal: That is, the *Focus* of the Radiant Point F is situated very nearly in the Plane CT. And since F is taken any how, the same will hold true of all the Points of the Plane EF, namely, that their *Foci* will be in the Plane CT. Which is demonstrated after the same manner in any other Case whatever. *Q. E. D.*

Corol.

Hence it follows, that the Image of the radiant Plane EF, to which the *Axis* of

of the *Lens* is Perpendicular, is alſo a Plane parallel to the former Plane: For the Image conſtituted in the Plane C T may be conſider'd, as a plane Surface ſending out Rays upon the ſecond Surface of the *Lens*. But if the Angle E A F be too great, ſo as that A T ſhall much exceed A C, and can by no means be look'd upon as equal to it, in that Caſe 'tis eaſy, after the manner of *Corol.* *Prop.* VIII. to determine whether the Image of the Plane E F will be convex or concave. For Example, it will be concave towards A, if B D the refracting Surface be of a denſer *Medium* and convex, or of a rarer *Medium* and concave, *& vice verſa.*

Prop. XVIII. Theor. VII.

THE Image appears from the Vertex of Emerſion under an Angle equal to that, under which the Object appears from the Vertex of Incidence. *Plate* II. *Fig.* 19.

Let G B the *Axis* of the *Lens* be ſuppoſed produced, and ſtanding perpendicularly on the radiant Plane F E H at E, which (by *Corol.* of the fore-going *Prop.*) is alſo Perpendicular to its Image *f* C *h*. Join the

the right Lines BF, BH, G*f*, G*h*. I say the Angles FBH, *f*G*h* are equal. Out of the innumerable Rays proceeding from the Point F, and after Refraction at the *Lens*, again united at *f* the Image of this Point, chuse two, one of which FB meets the *Lens* at the Vertex of Incidence B, and being there refracted, tends to φ the *Focus* of Transition of that Point, and being again refracted at L, is directed towards *f*: The other FD being first refracted in D, tends straight on to φ, till it emerges out of the *Lens* at the Vertex of Emersion, where being again refracted, it proceeds towards *f*.

DGB is the Angle of Incidence of the Ray DG, and CG*f* its Angle of Refraction; and LBG is the Angle of Incidence of the Ray LB, which (by *Corol.* 1. *Theor.* IV.) wou'd be refracted into BF, and EBF is its Angle of Refraction. Because the right Lines φB, φG (BG the Thickness of the *Lens* being neglected) become equal, the Sines of the Angles DGB, LBG which are proportional to these, and consequently the Angles themselves DGB, LBG, and therefore their Angles of Refraction, likewise CG*f*, EBF will be equal. After the same manner CG*h*, EBH are found equal: Therefore the Angles FBH, *f*G*h* are equal, which

which is ſhown after the ſame manner in any other Caſe whatever. *Q. E. D.*

Corol.

Hence it follows, that a radiant Line is to its Image made by a *Lens*, as the diſtance of that from the Vertex of Incidence, to the diſtance of this from the Vertex of Emerſion, or (the Thickneſs of the *Lens* being neglected) as their Diſtances from the *Lens*. But if the Radiant be a Surface, their homologous Lines will ſtill remain in the ſame Proportion; but the Radiant will be to its Image in a duplicate Proportion of thoſe Diſtances. Whence it will be eaſy to determine the Proportion which the *laſt Image* (which is immediately ſeen by the Eye) of an Object, made by the Mediation of one or more *Lens*'s, bears to the Object it ſelf.

From this *Prop.* it follows likewiſe, that a Radiant Line and its Image, are cut in the ſame Proportion by the *Axis* of the *Lens* produced.

PROP. XIX. PROB. XII.

THE *Position of a Radiant in the* Axis *of the* Lens *being given, to determine its Image made by a given* Lens, *with respect to an Eye placed in the* Axis *of the* Lens. *Plate* II. *Fig.* 20.

Let P C Q D be the *Lens* proposed, in whose *Axis* C D produced at Pleasure, suppose the Radiant to be placed, to the extreme Points of which the right Lines C E, C F do tend. Let *b* be the *Focus* of Rays proceeding from that Point of the Radiant which lies in the *Axis*, after Refraction at both the Surfaces of the *Lens*, found by *Prop.* XVI. Thro' which draw the Plane *e b f*, to which C D is Perpendicular. Then thro' D the right Lines D *f*, D *e* being drawn parallel to C F and C E, 'tis plain (from the two foregoing *Prop.*) that *f* is the Image of that Point to which the right Line C F is directed, and *e* is the Image of that to which C E is directed. From whence it is manifest, that the Image *f b e* will be seen by an Eye placed any where in C D produced beyond *b*, and receiving the *Rays* from the respective *Points* of the Image diverging.

But

But it is to be observ'd, that every Point of the Image f b e *does not, like the primary Radiant, send forth Rays every way and into all Parts; but the Rays of each Point constitute a Cone oppos'd at the Vertex to that Cone, which has the fore-mention'd Point for its Vertex, and the* Lens *that refracts the Rays for its Base*: Whence from the Situation of the Eye, and the Diameter of its Pupil being given, it will presently be known, whether the Eye will receive the Rays of any given Point, that is, whether it will see that Point. *Which Consideration must always take Place likewise, in Vision of Images made by a* Speculum.

For Example, if the Radiant be vastly distant, and the Surface P C Q plane, P D Q convex, and the *Lens* made of Glass with Air all around it, D *b* will (by *Corol.* 3. *Prop.* XIV.) be equal to the Diameter of the Sphere P D Q: For the Rays being parallel to BC, pass unrefracted thro' the plane Surface P C Q, upon which they fall perpendicularly; and the Image in respect to the Radiant is inverted.

But if P C Q be convex, and P D Q plane, C *b* will (by *Corol.* 1. *Prop.* XIII. and *Corol.* 1. *Prop.* XIV.) be equal to the Diameter of the Sphere P C Q, together with a Third Part of C D the Thickness of the *Lens*. And neglecting the Thick-

neſs of the *Lens*, as is uſually done in the object *Lens* of a Teleſcope, the Diſtance of the Image of a vaſtly diſtant Radiant, from a Plano-convex *Lens*, is equal to the Diameter of the Sphere.

If the Plano-convex *Lens* PQ were of Water, D *b* wou'd, by *Prop.* XIV. be ſeſquialteral of the Diameter.

If the *Lens* PQ be of Glaſs, and both its Surfaces equally Convex, the Thickneſs being neglected, D *b* will (by *Corol.* 1. *Prop.* XIV. and *Prop.* XV.) be equal to the *Semi-diameter* of either Sphere. In an entire Sphere of Glaſs, the Image of a very diſtant Radiant, will be at the diſtance of a Quarter of the Diameter behind the Sphere; for in this Caſe, the Thickneſs of the *Lens* cannot be neglected: In one of Water, this Diſtance equals the *Semi-diameter*.

In all theſe and the like Caſes, if the Sun be that diſtant Radiant, and the *Lens* be notably broader than the Image of the Sun, in the Place of the Image a Burning will be excited, and that more vehement than from a Concave *Speculum*, if the Image of the Sun be equally diſtant from a *Lens* and *Speculum* equally broad, becauſe of the greater loſs of Rays at this, than at that. If a Lucid Body be put in the fore-mention'd Place, the Image of it will be caſt at a very great Diſtance, and

and will enlighten thoſe Parts that are vaſtly remote.

If the Radiant be not vaſtly diſtant as before, but yet more remote from the *Lens*, than the Place of the Image of a vaſtly diſtant Radiant; beſides the Appearances juſt explain'd, namely, that the Image will be inverted, *&c.* if the Radiant approaches towards the *Lens*, the Image will recede, and *vice verſa*; till the Radiant comes to the place of the Image of a very diſtant Radiant, and then its Image will become vaſtly diſtant. All which may be ſeen in a darken'd Chamber, receiving no Light but thro' a convex *Lens*, upon which Radiants at different Diſtances caſt their Rays. The Place of the Image of any Radiant is known, by its being painted moſt diſtinctly upon a white unpoliſhed Plane, ſituated in the Chamber. Nor is there need of ſubjoining any Thing more of this Experiment, which is now very common; or of that other, founded upon the ſame optical Principles, in which by the pellucid Colours of a Picture painted upon Glaſs, and tranſmitting the cloſe Rays of a Flame, an Image is ſhown painted upon a white unpoliſh'd Plane.

If the Radiant be nearer the convex *Lens* than the Image of a vaſtly diſtant Radiant, then its Image will be form'd not on the oppoſite,

opposite, but on the same side of the *Lens*, and its Place according to what has gone before, from the Place of the Radiant being given will be determin'd. This Image is always erect, and greater than the Radiant: And as the Radiant approaches to the *Lens*, the Image likewise approaches to it, and in like manner they both recede from it at the same Time, but the Image more swiftly.

Fig. 21. If the convex *Lens* were chang'd into a concave One, the same Construction remains, and by a Calculation, (according to the *Corollaries* of *Prop.* XIV.) *e b f* the Image of a very distant Radiant, made by a Plano-concave *Lens* PQ, will be found erect, and on the same side of D with the Radiant, and distant from it by the *Diameter* of the Sphere P D Q. Now if the Surface P C Q be also Spherical, and the Radiant from being distant becomes near, its Image will be determin'd by the same Construction, if by *Prop.* XV. the Image of the radiant Point placed in the *Axis* of the *Lens* be first found. In which Case, besides what has been just now said, it is also to be observ'd, that the Image approaches to, or recedes from the *Lens* at the same Time as the Radiant does, but more slowly, as will be plain to any one, who will give himself the trouble of a Calculation.

Let

Let the Rays of any Radiant, inflected after ſuch a manner as to be ready to form the Image *e b f* if nothing hinder'd, be conceiv'd to be intercepted by a Plano-concave *Lens* P Q; upon whoſe concave Surface P D Q, deſcribed on a Diameter equal to the right Line D *b*, they firſt fall: The ſame Conſtruction as before, determines the Image made after the Refraction of theſe Rays at the forementon'd *Lens*. For (by *Corol.* 4. *Prop.* XV.) the Rays converging towards *b*, are ſo refracted from the concave Glaſs, as to become parallel to the *Axis*, that is, they conſtitute a vaſtly diſtant Image, to whoſe extreme Points (by the foregoing *Theor.*) the right Lines E C, F C or *e* D, *f* D are directed, and which is conſequently given in Poſition. And in this Caſe, the chief Thing to be obſerv'd is, that to an Eye placed about B, the diſtant Image appears inverted, in reſpect to the Image that wou'd have been made at *e b f*, without the Intervention of the *Lens*. It will be eaſy to proceed in all other Caſes, according to theſe Examples. *Q. E. F.*

PROP.

Prop. XX. Prob. XIII.

T*O find ſuch a Poſition of the Radiant with reſpect to a given* Lens, *that the Image made by the* Lens *may be equal to a given Figure, which is ſimilar to the Radiant: Or, which is the ſame Thing, that the Radiant may be to its Image made by the* Lens, *in a given Proportion. Plate* II. *Fig.* 22.

Let a *Lens* of Glaſs be given, for Example, let A B be the *Semi-diameter* of the firſt Surface, and C B the *Semi-diameter* of the ſecond Surface. Draw C M at pleaſure, making any Angle with C A. Let the Proportion of M D to D C be given, namely, that which the homologous Lines of the Radiant, and its Image bear to one another. Join A M, to which thro' B draw B E parallel, meeting the right Line C M in E; M E taken twice will be the ſought Diſtance of the Radiant from the *Lens*. If D *m* be taken in the ſame right Line, but on the other ſide of the Point D, equal to the right Line D M, and join A *m*; B *e* drawn parallel to this, will cut off another *m e*, which will likewiſe ſatisfy

tisfy the Problem: For the Radiant placed at the distance of twice *me* from the *Lens*, is to its Image (but then it will be made on the contrary side of the *Lens*, with respect to the former Image) as DM to DC.

By *Corol. Prop.* XVIII. The Radiant Line is to its Image made by a *Lens*, as their Distances from the *Lens*: But if twice ME or twice *me* be the distance of the radiant Point from the *Lens*, the distance of the Image of that Point from the *Lens*, will be to the former distance respectively, as MD to DC: As will be plain to any one, who will undertake a Calculation according to *Prop.* XV. For if AB be call'd a, BC b, the Thickness of the *Lens* C, MD r, and z the sought distance of the Radiant from the *Lens* of Glass about which Air is circumfused; we shall have $zz =$

$$\frac{6abz - 2acz \mp 6arz \times 2crz \mp 4acr}{3a - c + 3b}.$$

If the Thickness of the *Lens* be neglected (which is done in the Construction of the Problem) the sought Distance of the Radiant from the *Lens* will be equal to

$$\frac{2ab \times 2ar}{a+b}.$$

In

In the Scheme referr'd to, the *Lens* is made convex on both ſides; but the ſame Conſtruction will ſerve for any *Lens*, ſince from the variety of *Lens*'s, only the order of the Points A, B and C is chang'd. Another Conſtruction of this Problem is eaſily deduc'd from *Corol. Prop.* XVI.

Prop. XXI. Prob. XIV.

WITH two given Spherical Lens'*s or* Specula, *or one* Lens *and one* Speculum, *to make an Optical Machine, which to an Eye ſeeing at a given Diſtance, ſhall diſtinctly repreſent a given near Object under a given Angle, the Diſtance of the Eye from the laſt* Lens *or* Speculum *being likewiſe aſſign'd.*

Since the Eye is a Machine made on purpoſe, that the Images of exteriour Radiants may be diſtinctly painted upon its Bottom, (which is made concave for this end, as *Corol. Theor.* VI. requires,) all the reſt of its *Apparatus* conducing only to its Motion or Security, which are neceſſary to be provided for; it is plain, that a given Eye can only ſee diſtinctly at a given Diſtance from the Object. Now if the Eye cou'd place it

it ſelf at pleaſure, at ſuch a Diſtance from any Object as is neceſſary to diſtinct Viſion, (all other Things, as the Degree of Light, *&c.* being as they ſhou'd be) Viſion wou'd always be diſtinct. And tho' there be no Eye ſo ſtiff, as to ſee only at a determinate Diſtance, but can according to the flexibility or mobility of the Parts with which every Eye is endued, apply it ſelf to Objects placed at different Diſtances, and change its Figure according to the Diſtance given; ſo as to be no longer look'd upon as the ſame given Eye, but various and mutable, as Occaſion requires: Yet ſince this mobility is confined within certain Limits, and there are a great many Objects to which we cannot at Pleaſure come near enough to be within thoſe Limits; 'tis plain, there will be need of an Optical Machine to ſee them diſtinctly. But any Spherical *Lens* or *Speculum* will be ſufficient for this Purpoſe; ſince by its Aſſiſtance the Image of any Radiant (to which we cannot come ſo near as we wou'd) may be brought near us, (as is plain from what has been before demonſtrated) and then we ſhall be able to view it, ſince the Eye is ſuppoſed moveable at Pleaſure with reſpect to any Thing near.

But becauſe, beſides diſtinct Viſion, our Occaſions ſometimes require us to look into the more minute Parts of an Object;

and

and it is found by Experiment, that an Object seen under a leſs Angle than of one Minute, is conſider'd by the Obſerver as a Point, and its Parts not at all to be diſtinguiſh'd one from another: It often happens, that when the Object is brought nearer the Eye, that the Particles to be obſerv'd may be ſeen under a ſenſible Angle, and greater than the fore-mentioned one, the Object it ſelf becomes too near the Eye, and is out of the Limits required for diſtinct Viſion. This Inconveniency, if it be the only one, may be remedied by the Aſſiſtance of any given *Lens* or *Speculum*, by *Prop.* X. or XX. where the Image of an Object made by a given *Speculum* or *Lens* is repreſented in any given Meaſure.

But if both the fore-mention'd Inconveniences urge at the ſame Time, they are not to be removed without the help of two *Lens*'s or *Specula*, or one *Lens* and *Speculum*. Having ſhown therefore, what Aſſiſtance the Sight may receive from a ſingle given *Lens* or *Speculum*, we ſhall proceed to Machines made by two combined together; or to conſtruct the Problem propoſed univerſally. *Plate* II. *Fig.* 23, 24.

Let R be the given Object, S the given Angle in which it is to be repreſented, D the given Diſtance requiſite for diſtinct Viſion, and L the given Diſtance of the Eye from

from the *Lens*. Make the Triangle A O B, in which the Angle at the Vertex O is equal to the given S, and O E Perpendicular from the Vertex upon the Basis equal to the given D. If we take the middle Point of the Object placed in the *Axis* of the *Lens* or *Speculum*, which in Practice is very convenient, the Triangle A O B must be made Isosceles. Take O V equal to L. At V place either of the *Lens*'s or *Specula*, having its *Axis* in the right Line O E.

By *Prop.* VI. or XVI. having one *Focus* E of a *Lens* or *Speculum* given, find the other *e*; that is, that the Rays whose *Focus* before Incidence is *e*, may have E for their *Focus* after Inflection at the *Lens* or *Speculum*. Thro' *e* draw the right Line *a e b* parallel to the right Line A E B, meeting the right Lines V *a*, V *b* drawn thro' V parallel to the right Lines V A, V B, in *a* and *b*.

'Tis plain from *Prop.* IX. or XIX. that if *a e b* be the Radiant, A E B will be its Image: Wherefore if the given near object R, and another *Lens* or *Speculum* be placed after such a manner (by *Prop.* X. or XX.) that the Image of the Object R made by this *Lens* or *Speculum*, may obtain the Situation and Magnitude *a e b*; the Microscope required is made. For *a e b* is the Image of the Object R, the

 Image

Image of which Image, seen by the Eye placed in O, is A B B: Now this appears under the Angle A O B equal to the given S, and at the Distance O E equal to the given D, and consequently distinct; and the Distance of the Eye from the last *Lens* or *Speculum* in V, is equal to the given L. *Q.* E. F.

If the given Eye be an old Man's, every Thing else remaining as before, the right Line O E becomes infinite, and the Point *e* is found by *Prop.* III. or XIV.

If the Object proposed were vastly distant, the Problem wou'd be impossible. But if the Angle S were not given, a Telescope from the remaining *data* might thus be made. In the right Line given in Position, tending towards the propos'd distant Object, straight forward from O take O E, O V equal to D and L: And in V place one of the *Lens*'s or *Specula*, having its *Axis* in V E. By *Prop.* VI. or XVI. one *Focus* E of the *Lens* V being given, find the other *e*, which being supposed the *Focus* of Rays before Inflection at V, E may be their *Focus* after Inflection. In the right Line O E let the other *Lens* or *Speculum*, having its *Axis* in the same, be so placed, that the Image of the distant Object made by it, may be situated in the right Line *a e b* perpendicular to O E, and the Telescope required is made. For the

first

first Image of the distant Object is in *aeb:* And the Image of this Image is, by Construction, in A E B, whose distance from O is equal to the right Line O E or the given D, and consequently distinctly seen, and the Distance of the Eye from V is equal to the given L. But a distant Object being given, by *Prop.* XIX. its Image made by a given *Lens* or *Speculum*, and likewise the Image of that Image made by the given V, will be given, and consequently the Angle under which this last is seen by the Eye in a given Position.

But if O be an old Man's Eye, the Angle A O E, because of A B being in this Case vastly distant, is equal to A V B or *a*V *b*. Therefore the Angles under which a distant Object with and without a Machine appears, are as the Distances of the *Lens*'s or *Specula* from the common *Focus e*: For small Angles are almost as their Cotangents.

Scholium.

It will be very convenient that the *Lens* or *Speculum*, which immediately receives the Rays of the Object, and forms its first Image, (and is therefore call'd the *Object Glass*) be as perfect as possible. For the Errors or Defects of this *Lens*

or *Speculum* affect the Image made by it; and since this Image acts the Part of an Object, to be seen thro' the *Speculum* or *Lens* V, (which is nearest the Eye, and therefore call'd the *Ocular Glass*) its Defects, that is, the Defects or Errors of the Object Glass by which it is form'd, will be greater and more sensible, by how much the Image AEB is greater than *a e b*: That is, by how much more perfect (the Object Glass remaining the same) the Machine is made. But the Errors of the Ocular *Lens* or *Speculum* V, are equally sensible, whatsoever be the Image *a e b*, or the Object *Lens* or *Speculum* by which it is produced: That is, the Ocular Glass V remaining the same, its Defects are equally apparent and discoverable, to whatsoever Degree of Perfection the Machine, by changing the Object *Lens* or *Speculum*, which forms the Image *a e b*, be brought. For the Eye being given, the given Oculars *Lens* or *Speculum* V is always at the same Distances from the Image *a e b*, doing the Office of an Object and its Image AEB, and consequently shows the same Defects.

PROP.

Prop. XXII. Prob. XV.

WITH *Three or more given Spherical* Lens's *or* Specula, *to make a Machine, which to a given Eye shall distinctly represent a proposed distant Object under a given Angle, the Distance of the Eye from the last* Lens *or* Speculum *being assign'd.*

By the help of any one of the given *Lens*'s or *Specula* form the Image of the distant Object, and with the two others, by the foregoing *Prop.* make a Microscope that shall represent it in the Conditions proposed, and the Telescope required is made.

In like manner with these *Lens*'s or *Specula* may a Microscope be made, and then by adding a fourth, another Telescope: In all which we have the Proportion which the Image seen by the Eye bears to the Object, or which the Angle under which that is seen, bears to the Angle under which this is seen without the Machine: And consequently the Powers of a Machine in promoting Vision, are by *Corol. Prop.* IX. and XVIII. easily estimated.

Prop. XXIII. Prob. XVI.

To make a Spherical Lens *of Glass, whose Thickness is given, that shall to an Eye seeing at a given Distance, represent a given Object placed at a given Distance, under a given Angle distinctly, the Distance of the Eye from the* Lens *being likewise assign'd. Plate* III. *Fig.* 1.

We have hitherto shown what Assistance may be had from given *Lens*'s or *Specula*, or both, howsoever combined, in order to supply the Defects of Vision; it remains, that we demonstrate the manner of making a *Lens* for given Uses.

The radiant Line A *B*, whose Distance from the *Lens* is V E, is to be represented distinctly under the visual Angle *a* O *b*, to an Eye seeing distinctly at the Distance O *e*. In the Isosceles Triangle *a* O *b*, (whose base is *a b*, and Height *e* O) upon the Perpendicular O *e* (so situated as to bisect the radiant Line A B perpendicularly in E) take O L equal to the given Distance of the Eye from the *Lens*. Draw *a* L, *b* L, and produce them to D and F. Make L V equal

qual to the given Thickness of the *Lens.* From the right Line V L given in Position, perpendicular to the Glass at the Points of Incidence, draw the refracted Rays in Glass V α, V β, L H, L G corresponding to those in Air A V, B V, D L, F L. Produce the right Lines H L, G L, till they meet with the right Lines V β, V α in β and α. The right Line α β join'd, will cut the Perpendicular *e* O perpendicularly in *e*, because both the Angles of Incidence and of Refraction, on each side of the right Line O *e* are equal. From the *Foci* E and *e*, and the Vertex V being given, find (by *Corol.* 2. *Prop.* XV.) the Sphere of Glass K V M that may refract the Rays in Air diverging from E, so as to make them converge towards *e*. By the fore-mention'd *Corol.* find also N L P the Surface of a Sphere of Air passing thro' L, that may refract the Rays in Glass converging towards their *Focus* *e*, so as to make them afterwards diverge from *e*: The solid Figure M K N P, being made of Glass, and terminated by the Spherical Surfaces K V M, N L P and a Conic Surface whose *Axis* is V L, is the *Lens* required. But care must be taken, that the Portions V K, L N be not too great, because all the preceding Demonstrations hold only true of Rays falling near the Vertex.

By Construction, the Rays diverging from E, do after Refraction at the first Surface K V M, converge to the *Focus* ϵ: Wherefore (by *Prop.* XVII.) the Image of the radiant right Line A B (made by Refraction at K V M) is in the right Line $\alpha\beta$ given in Position: But the Ray A V, by Construction, after it is refracted at the Surface K V M comes to the Point α; therefore that Point α is the Image of the Point A, which is produced by the first Surface only. In like manner β is the Image of the Point B, and the right Line $\alpha\epsilon\beta$ is the Image of the radiant right Line A E B. Moreover (by Construction) the Rays within the *Lens* that converge towards ϵ, after they are refracted at the Surface N L P, diverge from *e*; wherefore (by *Prop.* XVII.) the Image of the future Image $\alpha\epsilon\beta$, made by Refraction at the Surface N L P, is in the right Line *a e b*: But the Ray G L within the *Lens*, proceeding directly towards α, after it is refracted, becomes L F, and proceeds from *a*; whence the Image of the future Image in α, after Refraction at N L P becomes *a*: And so likewise the Rays within the *Lens* that converge toward β, after Refraction at N L P diverge from *b*. That is, *a e b* is the Image of the Radiant A E B, placed at a given Distance V E from the *Lens*, made by the *Lens* NKMP,

NKMP, and to be ſeen diſtinctly by a given Eye in O, becauſe it is at the required Diſtance O *e* from the Eye, and is alſo ſeen under the given Angle *a* O *b* by the Eye O, which is at the given Diſtance O L from the *Lens*, and V L is the given Thickneſs of the *Lens*. *Q. E. F.*

The ſame Conſtruction ſerves, if the Eye of an old Man be given: For in that Caſe, *a e b* is at an infinite Diſtance, and the former right Lines *a* L, *b* L muſt be drawn thro' L parallel to the right Lines O *a*, O *b* given in Poſition, and the Center of the Surface N L P (by *Corol.* 6. *Prop.* XIV.) is diſtant from L by a third Part of the right Line L *ε*.

But if the Object propoſed be a diſtant one, the Conſtruction will become much more Simple: For the Center of the firſt Surface K V M (by *Corol.* 1. *Prop.* XIV.) will be diſtant from the Vertex V by a third Part of the right Line V *ε*.

Prop. XXIV. Prob. XVII.

To make a concave Spherical Speculum *of Glaſs, whoſe Thickneſs and Diameter of Concavity are given, ſo that the Rays*

Rays parallel to its Axis, *reflected from both its Surfaces, shall be collected in the same Point of its* Axis. *Plate* III. *Fig.* 2.

Because it is found by Experience, that *Specula* made of Metals, neither receive a due Figure and polishing so easily, nor preserve it so long, it will be convenient to use those of Glass. Let B then be the Vertex of the Concave Glass Surface EBF, A its Center, and BD the Thickness of the *Lens*, be given; we are to find DC the *Semi-diameter* of the Surface GDH, so that the Rays parallel to AB, and falling upon the Concave *Speculum*, as well those that are reflected from the Surface EBF, as those that are twice refracted at the fore-mention'd Surface EBF at their Entrance into it and Emersion out of it, but reflected from the Concave Surface GDH, may all meet in the same Point of the *Axis* AB: Or, which is the same Thing, a *Speculum* of Glass is to be made after such a manner, that the two Images of a distant Object made by the two Surfaces of the *Speculum*, may coincide, and consequently so as to be most powerful in burning, or in forming the Images of distant Objects, *Plate* III. *Fig.* 3.

From

From any Point L of the infinite right Line SM, on one ſide take LN equal to BD, and on the other ſide LR equal to twice BD, from N likewiſe directly forward take NT equal to thrice AB, and TV equal to twice NT, and VM alſo equal to twice RL. On the Center R, with the *Radius* RT deſcribe a Circle meeting the Perpendicular erected at L in K: Join MK, and at K erect the Perpendicular KS meeting the infinite right Line firſt drawn, in S. *Fig.* 2. In the right Line DA from D towards A, take DC equal to the right Line LS; and on the Center C with the *Radius* CD deſcribe an Arch of a Circle DG, ſimilar to the Arch BE deſcribed on the Center A, and join EG. The ſolid Figure generated by the Rotation of the plane Figure BDGE about the fix'd right Line BD, and conſiſting of Glaſs, is the *Speculum* required. *Q. E. F.*

Fig. 4. But if AB be ſufficiently large in reſpect to the Thickneſs BD, the Problem will admit of this more expeditious Conſtruction. Divide BD in O, ſo that the greater Segment BO may be to the leſſer OD, as 5 to 4; produce BA to C, ſo that CA may be equal to OD; and C will be

be very nearly the Center of the outward Surface G D H. *Q. E. I.*

We ſhall omit the Demonſtration of theſe two Conſtructions, becauſe it is very eaſily deduced from the following Analytical Calculation founded upon *Prop.* V. XIV. and XV. For if $AB = a$, and $BD = c$, we ſhall have $DC = \frac{9aa + 18ac + 5cc}{9a + 5c}$. In like manner may a *Speculum* be made, if the Radiant be in any other Poſition.

I had determined to have ſubjoin'd a general Calculation for finding the *Foci* of any *Speculum* or *Lens* univerſally: But that is abundantly done already for *Lens*'s by that excellent Analyſt *Edm. Halley*, in Philoſ. Tranſact. for the Month of *Nov.* 1693. and elegantly applied to particular Caſes.

Scholium.

Hitherto we have ſhown what Advantages may be expected from Spherical *Lens*'s or *Specula*, towards the Conſtruction of Machines: But the different Refrangibility of the Rays of Light, and that in given Rays given, immutable, and annex'd to certain Colours, diſcover'd

ver'd by that admirable Philoſopher Sir *ISAAC NEWTON*, has ſo much diſturb'd our Dioptrical Reaſonings, that no Exactneſs can now be hoped for from *Lens*'s, even tho' form'd into what Conoidical Figures we pleaſe. But ſince the Law of *Catoptrics* concerning the Equality of the Angles of Incidence and Reflection, is preſerv'd inviolable in Rays however heterogeneous, as the ſame great Man obſerves; it is better to uſe a *Speculum* inſtead of the Object *Lens*, which forming the Image of a diſtant Object at a conſiderable Diſtance, diſcovers the Errors that ariſe from the different Refrangibility of Rays ſenſible enough, and not at all to be diſſembled, if the Rays falling obliquely be admitted by an Aperture ſufficiently large, which is very often neceſſary: But in ſmaller *Lens*'s, ſuch as the ocular Ones, the Error is ſo ſmall and inſenſible, that they may be ſtill ſafely uſed.

James Gregory was the firſt who gave a Specimen of this ſort of *Cata-Dioptrical Teleſcopes* conſiſting of *Lens*'s and *Specula*, in *Optic. Promot. Prop.* 59. which was many Years afterwards given out by Mr. *Caſſegrain* a *French* Man, for his own. The ſame, upon Phyſical, as well as Geometrical

cal Accounts alter'd and improv'd, is publiſh'd by Sir *ISAAC NEWTON*, in his admirable *Theory of Lights and Colours*.

Since *Specula*, being opake Bodies, cannot have the ſame *Axis* with *Lens's* without being perforated at their Vertex, and conſequently ſuffering an irreparable Loſs (ariſing from both theſe Cauſes) of thoſe Rays that fall near the Vertex, and are moſt accurately reflected: The Poſition of the *Axis* may (by *Corol.* 1. *Prop.* II.) be alter'd as we pleaſe, by the help of a plane *Speculum*; and by this Means (beſides other Advantages) the Neceſſity of Perforation is quite taken away, and the Loſs of Rays falling near the *Axis* occaſion'd by the Opacity, is very much diminiſh'd by the Obliquity of the ſecond *Speculum*, which is obſerv'd by the Accurate *Newton* in his *Cata-Dioptrical* Teleſcope. But if by Reaſon of Phyſical Difficulties, in turning and poliſhing proper *Specula*, we muſt ſtill continue the uſe of *Lens's*, perhaps it wou'd be of Service to make the Object *Lens* of a different *Medium*, as we ſee done in the Fabrick of the Eye; where the Chryſtalline Humour (whoſe Power of refracting the Rays of Light

Light differs very little from that of Glaſs) is, by Nature, who never does any Thing in vain, join'd with the Aqueous and Vitreous Humours, (not differing from Water as to their Power of Refraction) in order that the Image may be painted as diſtinct as poſſible upon the Bottom of the Eye. There are likewiſe other Advantages of the fore-mentioned Artifice in the Animal Eye, which belong not to this Place.

FINIS.

ADDEN-

Light differs very little from that of Glass) is, by Nature, who never does any Thing in vain, join'd with the Aqueous and Vitreous Humours, (not differing from Water as to their Power of Refraction) in order that the Image may be painted as distinct as possible upon the Bottom of the Eye. There are likewise other Advantages of the fore-mentioned Artifice in the Animal Eye, which belong not to this Place.

FINIS.

ADDEN.

ADDENDA.

IN order to remove every Rub that might lie in the way of a Reader who is to begin with this Subject, and interrupt his Progreſs thro' this moſt excellent Treatiſe; beſides having all along in the Tranſlation explain'd ſuch Paſſages as the Author's Laconick Style has made ſomething too puzzling for a Beginner, and corrected ſeveral conſiderable Faults of the Preſs, which had eſcap'd his Care, I have thought it proper to ſubjoin a few Things, which may perhaps be of ſervice to obviate ſome farther Difficulties, and ſupply ſome ſeeming Defects.

I. From Prop. VI. Probl. V.

We may give a Solution of the following Catoptrical Problem, *for finding the* Focus *of any given* Speculum *universally; which the Author tells us, at the end of his Book, it was once his Design to have done.*

The Problem is this. *The* Focus, *or Point in the Axis of a given* Speculum, *from whence or towards which Rays proceed, being given, to find the* Focus *or Point where those Rays will again meet, after they fall upon, and are reflected by a given* Speculum.

Now this Problem being solved in one Case, namely that of Rays falling upon a *Convex Speculum*, and diverging from a certain Point in the Axis of the Speculum, will, *mutatis mutandis*, be applicable to all other Cases whatever, whether the *Speculum* be *Convex*, *Concave*, or *Plane*, and whether the Rays fall *diverging*, *converging*, or *parallel*.

[*Plate* 1. *Fig*. 9.] Let then BD be a given *convex Speculum*, whose Centre is A, and E a given Point in its Axis, from whence the Rays which fall upon the Speculum diverge; It is requir'd to find the *Focus*, or Point C, in which the Rays E D diverging from the *Focus*, or Point E, do, after reflection from the convex Speculum B D meet. Call E B the given Distance of the Point

Point E from B the Vertex of the Speculum d; AB the given Radius of the Speculum r; and BC the Diſtance of C the *Focus* required from B the Vertex of the Speculum x. By *Prop. VI. Probl. V.* AC $(r-x)$: CB (x) :: AE $(d+r)$: EB (d). From whence ariſes this Equation, $dr-dx=dx+rx$, which gives us BC $(x)=\frac{dr}{2d+r}$ for our general Rule; which ſhews that in the Caſe of *diverging Rays* falling upon a *convex Speculum*, the *Focus* C is always affirmative, and to be taken from the Vertex B directly forward: And that the greater d is, the greater will be the *Focal* Diſtance, till at laſt d becoming infinite, and conſequently the finite Term $+r$ vaniſhing, it will be $x=\frac{dr}{2d}=\frac{r}{2}$ or half the Radius: That is, the *Radiant* receding from a *convex Speculum*, the *Image* will alſo recede beyond the Vertex of the Speculum, but ſo ſlowly that when the Radiant becomes vaſtly diſtant, the Image will have got no farther than the middle Point, between the Vertex and Centre of the Speculum: And *vice verſa*, the nearer the Radiant approaches to the Speculum, the nearer the Image approaches to it, till at laſt they both meet and coinmide at the Vertex. According to *Obſervat.* 2. *Schol. Prop. VIII.*

If EB (d) be equal to AB (r), the *Fo-*

cal Diſtance B C becomes $= \frac{r}{3}$: but if E B (d) be equal to half A B ($\frac{1}{2}r$) the *Focal* Diſtance will in this Caſe become $= \frac{r}{4}$.

Now to apply this Univerſal Canon to all other poſſible Caſes; the Terms of which it conſiſts remaining always the ſame, 'tis only changing the Signs + or — according as the Caſe requires. For in the Caſe of *converging Rays*, the Point in the Axis of the Speculum towards which they converge, is on the other ſide of the Vertex of the Speculum, and conſequently its given Diſtance, in reſpect to the former, is *Negative*, or $-d$. For the ſame Reaſon, in the Caſe of a *concave Speculum*, the Centre of the Spherical Surface lying on the contrary ſide of the Vertex, the given Radius becomes *Negative*, or $-r$. And in theſe Caſes reſpectively, whereſoever the quantity d or r occurs, it muſt have a contrary Sign to what it had before.

If then *converging Rays* fall upon a *convex Speculum*, d being in this Caſe negative, the Rule will be $\frac{-dr}{-2d+r}$; which ſhews that when $2d$ exceeds r, the *Focus* will be ſtill affirmative, but if $2d$ be leſs than r, the *Focus* will be *negative*, or on the contrary ſide of the Vertex of the Speculum.

culum. That is, if the Point behind the Speculum, towards which the Rays converge, be at a greater Diſtance from the Vertex of the Speculum than half the Radius, the *Focus* is ſtill to be taken from B the Vertex directly forward, according to *Corol. Prop. IV.* But if its Diſtance be leſs than half the Radius, then the *Focus* muſt be taken from the Vertex B backwards, according to *Corol. Prop. VI.* When d is equal to $\frac{1}{2}r$, the *Focal* diſtance becomes $= \frac{-dr}{0}$, and is conſequently infinite. That is, Rays converging to a Point in the Axis of a convex Speculum, at an equal Diſtance between the Centre and Vertex, will after reflection proceed parallel, according to *Corol. Prop. III.* When d is equal to r, the *Focal* Diſtance becomes $= r$; that is, Rays converging towards the Centre of the Speculum are reflected by a convex Speculum back again upon themſelves.

If *parallel Rays* fall upon a *convex* Speculum, d in this Caſe becoming infinite, the Rule will be $\frac{dr}{2d} = \frac{1}{2}r$, which ſhews that Rays falling parallel upon a convex Speculum, are collected in a Point at the diſtance of half the Radius behind the Speculum, according to what has been demonſtrated at *Prop. III.* And conſequently a vaſtly diſtant Radiant will have its Image form'd in

in this Point; whence the Sun's Beams will be there collected, and heat or burn any thing placed therein.

If *diverging Rays* fall upon a *concave Speculum*, the Radius being in this Case $-r$, the Rule will be $\frac{-dr}{2d-r}$; which shews that when d is less than $\frac{1}{2}r$, the *Focus* is *affirmative*, when d is equal to $\frac{1}{2}r$ the *Focus* is *infinite*, and when d is greater than $\frac{1}{2}r$ the *Focus* is *negative*; and when d is equal to r, the *Focal* Distance is $= -r$. That is, if the Point in the Axis of the Speculum from which the Rays diverge, be nearer the Vertex than half the Radius, the *Focus* will still be behind the Speculum, according to *Prop. V.* If it be just at the Distance of half the Radius, the Rays after Reflection, will proceed parallel, according to *Prop. III.* If it be at a greater Distance than half the Radius, the *Focus* after Reflection, will be on the same side of the Speculum with the *Focus* before Reflection, or Point from which the Rays diverge. If it be at the distance of the whole Radius, the Rays after Reflection meet in the same Point from which they first diverge. It is moreover manifest, that the more d exceeds $\frac{1}{2}r$, the less will be the *negative Focal* Distance; but if d be infinite, the *Focal* Distance in this Case $= -\frac{dr}{2d} = \frac{1}{2}r$,

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can be no leſs than half the Radius: And on the contrary, the leſs d exceeds $\frac{1}{2}r$, the greater will be the *Focal* Diſtance; till at laſt, d becoming equal to $\frac{1}{2}r$, the *Focal* Diſtance, in this Caſe, $= \frac{-dr}{0}$, becomes infinite. But when d grows any thing leſs than $\frac{1}{2}r$, the *Focus* becoming *affirmative*, is thrown at a great diſtance on the contrary ſide of the *Speculum*; and by how much d is leſs than $\frac{1}{2}r$, the leſs will be this affirmative *Focal* Diſtance. So that a *Radiant* placed at a greater Diſtance than half the Radius from a *concave Speculum*, the farther it recedes from the Speculum, the nearer its *Image* which is on the ſame ſide approaches to the Speculum, and at the Centre of the Speculum they meet, and afterwards croſs one another, till the Radiant becoming *vaſtly diſtant*, the Image will come within half the Diſtance of the Radius from the Speculum: And *vice verſa*, the nearer the Radiant approaches the Speculum, the farther the Image recedes from it, and at the Centre of the Speculum both meet, and afterwards croſs one another, till at laſt the Radiant coming to half the Diſtance of the Radius from the Speculum, the Image becomes *vaſtly diſtant*: Whence if a lucid Body be placed at the diſtance of half the Radius from a concave Speculum, it will enlighten Places that are vaſtly diſtant. If the Radiant comes nearer the Speculum than

than half the Radius, the Image will be caſt from before the Speculum to a great diſtance on the contrary ſide; and the nearer the Radiant now approaches to the Speculum, the nearer will the Image likewiſe approach to it, till at laſt they both coincide at its Vertex, and *vice verſa*, according to *Obſervat.* 1. *Schol. Prop. VIII.*

If *converging Rays* fall upon a *concave Speculum*, *d* and *r* being in this Caſe both negative, the Rule will be $\frac{dr}{-2d-r}$; which ſhows that the *Focus* is always *negative*. That is, Rays that fall converging upon a concave Speculum, will always be collected in a Point or *Focus* on this ſide the Speculum; according to *Corol. Prop.* V. If *d* be equal to *r*, the *Focal* Diſtance is $=\frac{-r}{3}$; but if *d* be equal to $\frac{1}{2}r$, the *Focal* Diſtance is $=\frac{-r}{4}$.

If *parallel Rays* fall upon a *concave Speculum*, *d* in this Caſe becoming infinite, the Rule will be $\frac{-dr}{2d}=-\frac{1}{2}r$. That is, Rays falling Parallel upon a concave Speculum, are collected in a Point at the diſtance of half the Radius on this ſide the Speculum,

culum, as has been demonſtrated at *Prop. III.* Whence if the Sun's Beams be receiv'd upon ſuch a *Speculum*, the ſame will be the burning Point.

If *diverging Rays* fall upon a *plane Speculum*, the Radius *r* being infinite, the Rule will be $\frac{dr}{r} = d$; That is, the *Focus* of diverging Rays reflected by a Plane *Speculum*, will be at as great a diſtance behind the *Speculum*, as the Point from which they diverge is before the *Speculum*; according to *Prop. II.* And conſequently the Image of any Radiant, made by Reflection from a Plane *Speculum* will be ſeen as far behind the *Speculum* as the Radiant is before the *Speculum*; and they will both not only recede from, and approach to the *Speculum* at the ſame time, but likewiſe keep an equal pace with one another.

If *converging Rayss* fall upon a *plane Speculum*, *d* being negative, and *r* infinite, the Rule will be $\frac{-dr}{r} = -d$; that is, Rays converging to a Point at a certain Diſtance behind a Plane *Speculum*, will have their *Focus* at an equal Diſtance before the *Speculum*; according to *Corol.* 1. *Prop. II.*

If *parallel Rays* fall upon a *plane Speculum*, both *d* and *r* being in this caſe infinite,

finite, the *Focal* Distance $\frac{dr}{2d \div r}$ will be also infinite. That is, Rays falling parallel upon a Plane *Speculum*, will be reflected back parallel.

It is worth observing, that the consideration of *diverging Rays* relates to Objects that are near us, and such as we examine with our naked Eye, or by the help of a Microscope: *Parallel Rays* are consider'd when we have to do with Objects vastly distant, and such as we look at thro' Telescopes. And *converging Rays* fall under our Consideration, when the Rays proceeding diverging or parallel from any Object are by one Speculum or *Lens* made to converge, and then intercepted by the Interposition of another Speculum or *Lens*, before they arrive at their Point of Convergence; which is of great use in examining the Effects of Optical Machines, made by a Combination of more than one *Lens* or Speculum, and constructing such as are proper for any assign'd purpose, where this Contrivance is often absolutely necessary.

There are two farther *Uses* of this Method, the first is, to determine what degree of *Convexity* or *Concavity* is necessary for a *Speculum* to represent a given Object at a given *Focus*. And this is very easily done from the Equation first found $dr - dx = dx \div rx$: For d and x being given; the Radius of Convexity

Convexity $r = \frac{2dx}{d-x}$. Where it is plain, that if x be greater than d, r will be a negative Quantity, and the Problem impoſſible for a convex *Speculum*. That is, if it be required to repreſent a given Object, at ſuch a given *Focus*, whoſe Diſtance on the other ſide of the *Speculum* ſhall be greater than the Diſtance of the Object on this ſide; inſtead of a convex Glaſs, we muſt uſe a concave, whoſe negative Radius will be $= \frac{2dx}{x-d}$. After the ſame manner the Degree of Concavity is found from the ſame Equation, only changing the Sign of r from $+$ to $-$: For d and x being given, we ſhall have the Radius of Concavity $r = \frac{2dx}{x-d}$. Where it is manifeſt, that if d be greater than x, r will be a negative Quantity, and the Problem impoſſible for a concave *Speculum*. That is, if it be required to repreſent a given Object, at ſuch a given *Focus*, whoſe Diſtance on the other ſide of the *Speculum*, ſhall be leſs than the Diſtance of the Object on this ſide, inſtead of a concave *Speculum*, we muſt make uſe of a convex, whoſe affirmative Radius is $= \frac{2dx}{d-x}$. And in both Caſes if d be equal to x, then the Radius either of Convexity or Concavity $= \frac{2dx}{\underset{-}{+}d \underset{+}{-} x} = \frac{2dx}{0}$, will be infinite, and

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the Problem will be impoſſible for either a convex or concave Speculum. That is, if it be required that a given Object ſhall be repreſented at ſuch a given *Focus*, whoſe Diſtance on the other ſide of the Speculum ſhall be equal to the Diſtance of the Object on this ſide, inſtead of a *convex* or *concave* Speculum, a *palne* Speculum is the only one that can be uſed. If d be infinite, the Radius of Convexity becomes $\frac{2dx}{d} = 2x$; but the Radius of Concavity $\frac{2dx}{-d} = -2x$, will have a negative Value, which ſhews that the Problem is impoſſible in the Caſe of a concave Speculum, That is, a vaſtly diſtant Object cannot be repreſented at any given *Focus* behind a concave Speculum; but may by a Convex, whoſe Radius of Convexity muſt be equal to twice the given *Focal* Diſtance behind the Speculum. If x be infinite, then the Radius of Convexity $= \frac{2dx}{-x} = -2d$, has a negative Value, and the Problem is impoſſible for a convex Speculum; but the Radius of Concavity is $= \frac{2dx}{x} = 2d$: Whence if we would have a given Object repreſented at an infinite Diſtance behind the Speculum, we can make uſe of none but a concave Speculum, whoſe Radius

Radius of Concavity muſt be equal to twice the given Diſtance of the Object.

Hitherto we have conſider'd the *Focus* as affirmative, that is, behind or beyond the Speculum; but the ſame Rule, only changing the Sign of x in the Equation $x = \frac{dr}{2d+r}$, will equally hold if we would have the *Focus* negative, or on the ſame ſide of the Speculum with the Object. For in the Caſe of a convex Speculum we ſhall have the Radius of Convexity $= \frac{-2dx}{d+x}$ always of a negative Value, and conſequently the Problem is always impoſſible for a convex Speculum; but in the Caſe of a concave Speculum, the Radius of Concavity will be $= \frac{-2dx}{-d-x}$; which ſhews that the Problem is always poſſible for a Concave Speculum, be the quantities d and x as they will. If we would have $d = x$, then the Radius of Concavity will be alſo equal to d or x: That is a Concave Speculum, whoſe Radius is equal to the diſtance of the Object from the Speculum, will reflect the Image into the ſame place with the Object.

The other *Uſe* of this Method is, from the *Image* and *Speculum* given, to find the *Diſtance* of the Object from the *Speculum*. That is in the Equation firſt found

dr

$dr - dx = dx + rx$, x and r being given, we are to find d, which will consequently be $= \frac{rx}{r - 2x}$, in *convex Specula*, and in *concave* $= \frac{rx}{r + 2x}$. Whence 'tis plain that in *convex Specula*, the Problem will be impossible, when x exceeds $\frac{1}{2} r$, but in *concave Specula* it will always be possible. That is, if the Image is to be at a greater distance than $\frac{1}{2}$ the Radius behind the Speculum, it cannot be made by an Object placed before a convex Speculum at what distance soever: But let the Distance of the Image behind the *Speculum* be what it will, it may be form'd by an Object exposed at some certain distance before a concave Speculum. If the given *Focus* be negative, or the Image on the same side of the Speculum with the Object, then changing the Sign of x, in the forementioned Equation, we shall have, in the Case of *convex Specula*, $d = \frac{-rx}{r + 2x}$, and in the Case of *concave Specula* $d = \frac{-rx}{r - 2x}$. So that the Problem will always be impossible for *convex Specula*, and only possible for *concave* ones, when $\frac{1}{2} r$ does not exceed x, or when the Image is not nearer the Speculum than by half the Radius.

II.

II. From Corol. Prop. IX.

WE *may deduce a Solution of the following* Catoptrical Problem, *of magnifying or diminishing a given Object by a given Speculum in any assign'd Proportion.*

The Problem is this; *To find at what Distance from a given Speculum it is necessary to place an Object, in order that the Homologous Lines of the Image made by the Speculum, may bear any assign'd proportion to those of the Object.*

Since it is evident from this *Corol.* that the Homologous Lines of the Radiant and Image, are to one another as their Distances from the Speculum respectively: It follows, That if b to c express the Proportion which the Homologous Lines of the Object and Image are to bear to one another, b will be to c, as d the Distance of the Object to x the *Focal* Distance of the Image. Whence if we compare this value of x with that deliver'd in the foregoing Problem, we shall have $\frac{cd}{b} = \frac{dr}{2d+r}$. And consequently the Distance required $d = \frac{br - cr}{2c}$, will be our general Rule; and will, *mutatis mutandis,*

tandis, extend it ſelf to all poſſible Caſes whatever, tho' in its preſent Form, it regards the Caſe of *convex Specula* in particular. For *concave Specula* 'twill ſtand thus $d = \frac{cr - br}{2c}$. If the Image be deſired on the ſame ſide of the Speculum with the Object, x being negative, in the Caſe of *convex Specula*, 'twill be $d = \frac{-br - cr}{2c}$; and in the Caſe of concave $d = \frac{br + cr}{2c}$.

From whence 'tis plain that there is no magnifying an Object by a convex Speculum, for c being in this Caſe greater than b, the Rule for the affirmative *Focus* $d = \frac{br - cr}{2c}$ will have a negative Value, and that for the negative *Focus* $d = \frac{-br - cr}{2c}$ has always a negative Value: So that we can only diminiſh an Object, and make it appear leſs, by a convex Speculum, and that only when the *Focus* is affirmative, or the Image to be repreſented behind the Speculum. And by a concave Speculum there is no diminiſhing an Object, as long as the *Focus* is affirmative, for b being greater than c, the Rule in that Caſe $d = \frac{cr - br}{2c}$ will have a nega-

tive

tive Value; ſo that we can only magnifie an Object, and make it appear greater behind a concave Speculum. But if the *Focus* be negative, and the Image and Object to be both on the ſame ſide of the Speculum, the Rule being $d = \frac{br + cr}{2c}$, ſhews that in this Caſe a concave Speculum will magnifie or diminiſh an Object in what proportion we pleaſe.

It is to be obſerv'd, that if the Object be a Right Line, the Proportion b to c will expreſs the Proportion between the Object and Image themſelves; but if the Object be a Plane Figure, the Proportion b to c will be only Subduplicate of that which the Object bears to the Image, as we learn from *Euclid*. So that if b to c be as 2 to 1, the Object and Image will be as the Squares of theſe Numbers, or as 4 to 1. But it muſt alſo be noted, That Painters uſually meaſure the largeneſs of their Figures by the ſimple Proportion of their homologous Lines; ſo that when they ſpeak of an Human Figure twice as big as the Life, their meaning is that the Homologous Lines of this Figure are twice as great as thoſe of the Life; or that the Dimenſions of every Member in length and breadth are twice as great as thoſe of the Man repreſented; tho' properly ſpeaking, the Picture is four times as big as the Life.

D If

If the out Lines of the Image be desired twice as big as the Life, and the *Focus* affirmative, *c* being in this case greater than *b*, the Problem will be impossible for any convex Speculum; but the Object being placed before a concave Speculum at the Distance $\frac{cr - br}{2c} = \frac{r}{4}$, or $\frac{1}{4}$ of the Radius, will have its Image magnified in the Proportion assign'd. If we would have the *Focus* negative, and the Image represented on the same side of the Speculum with the Object, still the Problem will be impossible for a convex Speculum; but if the Object be placed before a concave Speculum at the Distance $\frac{br + cr}{2c} = \frac{3r}{4}$, or $\frac{3}{4}$ of the Radius, its Image made by the Speculum will be magnified in the Proportion assign'd.

Thus let the Proportion *b* to *c* be what it will, the Rule will always give us the Distance, at which the Object must be placed before the given Speculum, in order to have its Image magnified, or diminished in that Proportion. I shall only add one Instance more, and that is, supposing *b* and *c* were equal, and the *Focus* affirmative: In this Case we should have both for convex and concave *Specula* $d = \frac{\div br \mp cr}{2c} = \frac{0}{2c} = 0$: That is, the Object must be placed at the very

very Vertex of the Speculum, in which Caſe we know both Object and Image coincide. If the *Focus* were negative, then no convex Speculum will do; and the Rule for *concave Specula* will be $d = \frac{br + cr}{2c} = \frac{2cr}{2c} = r$: That is, the Object muſt be placed in the Centre of the Speculum.

There are two farther *Uſes* to be made of this Method; the firſt is, The Diſtance at which the Object is to be placed before the Speculum, and the Proportion in which the Image is to be magnified, or diminiſhed, being given, to find what Degree of Convexity or Concavity the Speculum ſhould have, in order to magnifie or diminiſh the Image in the Proportion aſſign'd. That is, in the Rule $d = \frac{br - cr}{2c}$, d, b, and c being given, we are required to find r; which will give us in the Caſe of a convex Speculum, and an affirmative *Focus*, $r = \frac{2cd}{b-c}$; and in the Caſe of a concave Speculum, $r = \frac{2cd}{c-b}$. If the *Focus* be negative, for convex Specula, the Rule ſtands $r = \frac{2cd}{-b-c}$; for concave, $r = \frac{2cd}{b+c}$.

From all which it appears, that if c be greater than b, or if the Image be desired greater than the Object, and to be represented behind the Speculum, no Convex will do, but a Concave will, whose Radius is $\frac{2cd}{c-b}$. As likewise, if b be greater than c, or the Image be desired less than the Object, and to be represented behind the Speculum, no Concave will do, but a Convex will, whose Radius must be $\frac{2cd}{b-c}$. If the *Focus* be required negative, or the Image to be on the same side of the Speculum with the Object; the Problem is altogether impossible for a convex Speculum, whether to magnifie or diminish; and always possible for a concave Speculum either to magnifie or diminish.

The other *Use* we may make of this Method, is from the Distance of the Object before the Speculum, and the Radius of Convexity or Concavity being given, to find the Proportion b to c, which the Object will bear to its Image made by the given Speculum. That is, in the forementioned Rule, having d and r given, to find the Proportion of b to c. Whence in the Case of a convex Speculum, and an affirmative *Focus*, 'twill be $b . c :: 2d + rr$; And in the Case of a Concave, $b . c :: r - 2d . r$. But if

if the *Focus* be negative, in *convex Specula*, 'twill be $b . c :: -2d - r . r$; and in *concave*, $b . c :: 2d - r . r$.

So then in the Caſe of a convex Speculum, and the Image behind the Speculum, b will always be greater than c, becauſe $2d + r$ is of neceſſity greater than r: And in the Caſe of a Concave, b will always be leſs than c for the like reaſon; and if d be equal to $\frac{1}{2} r$, then c will be infinite in reſpect to b. But if the Image be required on the ſame ſide of the Speculum with the Object, in *convex Specula*, 'twill be found always impoſſible: And in *concave*, poſſible in all Caſes whatever, both of magnifying and diminiſhing; for if d be greater than r, than b will be greater than c; if d be leſs than r, b will be leſs than c; and if d be equal to $\frac{1}{2} r$, then c will be infinitely greater than b.

III. At Corol. 3. Prop. XV.

THE *Author gives a Construction to find the* Focus *of Rays refracted at a Spherical Surface, and towards the latter end of that* Corol. *applies the same to the Case of a* plane *Surface: Affirming, that in this Case the Right Line* FM, *which determines the* Focus *by its meeting the Axis* BA *produced somewhere in* C, *will be the same with a Right Line, joining the Point* F *with another Point taken in* BE *produced, at such a Distance from* B, *as to make that Distance bear the same Proportion to* BE *which* AN *does to* AM. *Because the Tangents of Angles are reciprocally as their Cotangents.*

[*Plat. III. Fig.* 5.] That this may be more easily conceiv'd, Suppose the Surface BD plane, and take the Point C upon BE produced, so as that BC may be to BE, as AN

AN to AM, and join FC. *We are to ſhew that the Right Line FM produced, will in this Caſe meet the Axis AB produced, in the Point C, and there determine the Focus.* Join EN, EM, and on the Centre E with any Radius, as EA, deſcribe an Arch of a Circle AG, and draw the Tangents of the Angles AEN, AEM, which will always be as AN to AM, and are in the preſent Caſe thoſe very Lines, as likewiſe their reſpective Cotangents GH, GK. Since A the Centre of the plane Surface BD, and conſequently the Right Line AMN is at an infinite diſtance from B, the Right Line FM becomes parallel to EM; and conſequently if produced beyond F, will meet the Axis AB produced ſomewhere, ſuppoſe at C, ſo as to make the Triangle CBF ſimilar to the Triangle KGE. And therefore the right Line EH, paſſing thro' the Vertex of both thoſe Triangles, will cut their Baſes CB, GK ſimilarly in E and H, ſo as to make BC to BE, as GK to GH. But becauſe the Tangents of Angles are reciprocally as their Cotangents, AN is to AM, as GK to GH; therefore BC is to BE as AN to AM. And conſequently the Point C, where the right Line FM produced meets the Axis AB produced, is that very Point C taken at firſt upon BE produced, ſo as that BC may be to BE as AN to AM. *Q. E. D.*

IV.

IV. At Prop. XVI. Probl. XI.

THE *Author recommends the use of* Analytical Calculations, *for finding the* Foci *of* Lens's, *as far better than the very nicest* Geometrical Constructions. *For which reason it cannot be improper to give the less skilful Reader an Example, that he may see how such* Calculations *are to be manag'd.*

[*Plat. III. Fig. 6.*] Suppose BD to be the given *Lens*, and E a Point in its Axis, from which Rays diverging fall upon the *Lens*, A the Centre of its first Spherical Surface, and C the Centre of its second Spherical Surface, BD the thickness of the *Lens*, and I to R the *Ratio* of Incidence to Refraction. And it is requir'd to find F the *Focus* of those Rays after Refraction at both Surfaces of the *Lens*. We must first find f the *Focus* of those Rays after their Refraction at the first Surface only, or their *Focus* of *Transition*. Call EB d, BD t, AB r, CD ϱ, Bf x, DF y. By *Proposit. XV.* EA $(d+r)$: Af $(x-r)$ $+f$B(x): BE (d)

(d) : : I : R : Whence multiplying the Extremes and Means, $Rdx + Rrx = Idx - Idr$; and Bf $(x) = \frac{Idr}{Id - Rd - Rr}$. Whence it is plain, that if d be so great in respect to r, that Id exceeds Rd + Rr, the *Focus* f is affirmative, and to be taken from B the Vertex of the refracting Surface directly forward, as at *Fig.* 6: If Id is less than Rd + Rr the *Focus* f is negative, and to be taken from B backwards, as at *Fig.* 7; and if Id be equal to Rd + Rr, Bf becomes infinite, and the Rays proceed parallel.

Thus having found f the *Focus*, after the first Refraction, we may by the same means find F the *Focus* after the second Refraction. For by the same *Prop. XV.* fC $\left(\frac{Idr}{Id - Rd - Rr} - t + \varrho = \frac{Idr - Idt + Rdt + Rrt + Id\varrho - Rd\varrho - Rr\varrho}{Id - Rd - Rr}\right)$: CF $(y + \varrho)$ + FD (y) : Df $\left(\frac{Idr}{Id - Rd - Rr} - t = \frac{Idr - Idt + Rdt + Rrt}{Id - Rd - Rr}\right)$: : R : I : (I to R at the Emersion of Rays from any Lens, being as R was to I at their Immersion into it.) Wherefore multiplying the Extremes and Means, we have

$I^2 dry - I^2 dty + IRdty + IRrty + I^2 d\rho y - IRd\rho y - IRr\rho y = IRdry - IRdty + R^2 dty + R^2 rty + IRdr\rho - IRdt\rho + R^2 dt\rho + R^2 rt\rho$; And consequently DF. ($y$) =

$$\frac{IRdr\rho - IRdt\rho + R^2 dt\rho + R^2 rt\rho}{I^2 dr - IRdr - I^2 dt + 2IRdt - R^2 dt + IRrt - R^2 rt + I^2 d\rho - IRd\rho - IRr\rho}.$$

Which Equation if we put $p = \frac{R}{I - R}$, may be abridged, and reduced to DF (y) =

$$\frac{Ipdr\rho - Rd\rho t + Rpr\rho t}{Idr - Idt + Rrt + Id\rho + Rdt - Ipr\rho}.$$

And is evidently the ſame with that, which the famous Dr. *Halley* has given long ago, in the *Philoſophical Tranſactions*, *for finding the Foci of Optical Glaſſes univerſally.*

This Calculation being general, will ſerve for all ſort of *Lens*'s, be the Matter of which they are made, and the Ambient *Medium* what they will, or what ever be the *Ratio* of I to R: And tho' it is made for *Lens*'s whoſe Surfaces are both *convex*, yet *mutatis mutandis*, it will extend to *Lens*'s of any other Figure whatever, whether *double-convex* or *double-concave*, *plano-convex* or *plano-concave*, or *convexo-concave*, which laſt ſort are commonly call'd *Meniſci*. For the Radius of a *concave* Surface being on the contrary ſide, or *negative* with reſpect to that of a *convex*, and the Radius of a *plane* Surface *infinite*: 'tis only changing all

all the Signs + or — with which the Radius of the respective Surface, which we would have *concave* instead of convex, is affected in the general Rule; or making all the Terms infinite, which involve the Radius of the respective Surface, which we would have *plane* instead of convex. So likewise if we would have it extend to other Rays besides *diverging* ones; the distance of the Point where *converging* Rays meet, from the first Surface of the *Lens*, being on the contrary side or *negative*, in respect to that of *diverging* Rays, and the distance of the Point where *parallel* Rays meet, from the same Surface, being *infinite*: 'Tis only changing the Signs of all the Terms where we meet with d, if the Rays are supposed *converging*; or making those same Terms infinite if the Rays are supposed *parallel*.

In the Case of a *double-Convex* of Glass, if the ambient *Medium* be Air, I being to R as 3 to 2, we shall have the *Focal* Distance from the second Surface of the *Lens*, $y = \frac{6dr\rho - 2d\rho t + 4r\rho t}{3dr - dt + 2rt + 3d\rho - 6r\rho}$: if the ambient *Medium* be Water, I being to R as 9 to 8, the Rule will be $y = \frac{72dr\rho - 8d\rho t + 64r\rho t}{9dr - dt + 8rt + 9d\rho - 72r\rho}$. For a *double-Convex* of Water, and the ambient

Medium of Air, I being to R as 4 to 3, the Rule is $y = \frac{12dr\rho - 3d\rho t + 9r\rho t}{4dr - dt + 3rt + 4d\rho - 12r\rho}$. And for a *double-convex* of Diamond, in a *Medium* of Air, I being to R as 5 to 2, the Rule would be $y = \frac{\frac{10}{3}dr\rho - 2d\rho t + \frac{4}{3}r\rho t}{5dr - 3dt + 2rt + 5d\rho - \frac{10}{3}r\rho}$,

If the thickneſs of the *Lens* be neglected, which is generally not conſiderable, the Terms where t occurs being rejected, the Rule is ſtill farther abridg'd to $y = \frac{pdr\rho}{dr + d\rho - pr\rho}$. Where it is evident, that if d be ſo ſmall in reſpect to r and ρ, that $dr + d\rho$ is leſs than $pr\rho$, the *Focal* Diſtance y will be negative, and $= \frac{pdr\rho}{-dr - d\rho + pr\rho}$, or the Rays after the two Refractions at both Surfaces of the *Lens*, will ſtill proceed diverging from ſome Point, before the ſecond Surface of the *Lens*; and if $dr + d\rho$ be equal to $pr\rho$, y is infinite, and the Rays after Emerſion from the *Lens* proceed parellel. The Errour in neglecting t is ſo ſmall, that if for the eaſe of the Calculation, we ſuppoſe a *Lens* of Glaſs equally *convex* on both ſides, and expos'd to *parallel Rays*; r being in this caſe equal to ρ, and d infinite, the *Focal* Diſtance when t is neglected is $\frac{2drr}{2dr} = r$, and when it is conſider'd

$\frac{6rr - 2rt}{6r - t}$; which is only $\frac{rt}{6r - t}$ or nearly $\frac{1}{6}t$ leſs than the former. In the caſe of *converging Rays* falling upon a *double-convex* of Glaſs, we have $y = \frac{-2dr\rho}{-dr - d\rho - 2r\rho}$, always affirmative: And if the Rays are *parallel*, d being infinite, 'twill be $y = \frac{2dr\rho}{dr + d\rho} = \frac{2r\rho}{r + \rho}$, which alſo gives the *Focus* always affirmative, or behind the *Lens*. *Diverging Rays* falling upon a *double-concave*, give $y = \frac{2dr\rho}{-dr - d\rho - 2r\rho}$ always negative, as in the caſe of *converging Rays* on a *double-convex* 'twas always affirmative: But if the Rays are *converging*, it will be $y = \frac{-2dr\rho}{dr + d\rho - 2r\rho}$ affirmative, when $dr + d\rho$ is leſs than $2r\rho$, or when the *Focus* of *diverging Rays* collected by a *double convex* is negative, and *vice verſa*; If the Rays are *parallel*, 'tis $y = \frac{2dr\rho}{-dr - d\rho} = \frac{2r\rho}{-r - \rho}$, always negative. A *plano-Convex* Glaſs, the *plane* Surface being expoſed to *diverging* Rays, gives, r being infinite, $y = \frac{2dr\rho}{dr - 2r\rho} = \frac{2d\rho}{d - 2\rho}$, affirmative or negative, according as d is greater

ter or leſs than 2ρ; if expoſed to *converging* Rays, $y = \frac{-2d\rho}{-d-2\rho}$, always affirmative; if to *parallel* Rays, $y = \frac{2d\rho}{d} = 2\rho$, ſo that the Image of a vaſtly diſtant Object is always form'd by a *plano-Convex Lens*, the plane ſide being turn'd towards the Object, juſt at the Diſtance of the Diameter of the ſecond Surface behind it. A *plano-concave Lens* expoſed on the *plane* ſide to diverging Rays, gives $y = \frac{-2dr\rho}{dr+2r\rho} = \frac{-2d\rho}{d+2\rho}$, always negative; to converging Rays, $y = \frac{2d\rho}{-d+2\rho}$, affirmative, when the *Focus* of diverging Rays on a *plano-Convex* is negative, and *vice verſa*; to *parallel Rays*, $y = \frac{-2d\rho}{d} = -2\rho$, ſo that the Image of a vaſtly diſtant Object, is always form'd by a *plano-Concave* at the diſtance of the Diameter before the ſecond Surface, as it is by a *plano-Convex* behind. A *Meniſcus* expos'd on the *concave* ſide to *diverging* Rays, gives $y = \frac{-2dr\rho}{-dr+d\rho+2r\rho}$, affirmative only when d and r are ſo great in reſpect to ρ, that dr exceeds $d\rho + 2r\rho$: To *converging Rays*, $y = \frac{2dr\rho}{dr-d\rho+2r\rho}$, affirmative or negative,

gative, according as $dr + 2r\rho$ is greater or less than $d\rho$: If to *parallel* Rays $y = \frac{-2dr\rho}{-dr + d\rho} = \frac{-2r\rho}{-r + \rho}$, affirmative, if the Radius of *concavity* is greater than the Radius of *convexity*; and negative if less; and infinite if equal, for the Effects of the first Surface are, in that case, exactly destroy'd by the second, and the Rays suffer'd to proceed still parallel.

It is to be observ'd, that if the thickness of the *Lens* is neglected, as inconsiderable, the *Focus* of all sorts of Rays falling upon any *Lens* will be exactly the same, upon which soever Surface of the *Lens* they are first receiv'd. But if the thickness of the *Lens* be consider'd, there will be some difference in the *Focal* Distance, according as you turn this or that Surface of the *Lens* towards the Rays. And this difference is easily found from the general Rule: For upon turning the other Surface of the *Lens* towards the Rays, ρ becomes r, and r changes to ρ; by which means the Rule will give us the *Focal* Distance in both Cases, and substracting one from the other, we find their Difference. Thus, if, to abridge the Rule, we suppose the Rays parallel, d being infinite, we have for a *double-Convex* in one case $y = \frac{\mathrm{I}pr\rho - \mathrm{R}\rho t}{\mathrm{I}r - \mathrm{I}t + \mathrm{I}\rho + \mathrm{R}t}$, and up-

on

on turning the *Lens*, $y = \frac{I\rho r - Rrt}{I\rho - It + Ir + Rt}$; wherefore ſubſtracting one from the other, according as ρ or r is greateſt, we ſhall have the difference in *double Convex*, occaſion'd

by turning the *Lens* $= \frac{\pm Rrt \mp R\rho t}{Ir \mp It \pm Rt + I\rho}$,

or in Glaſs $\frac{\pm 2rt \mp 2\rho t}{3r - t + 3\rho}$. And this is applicable to *Lens*'s of any other Figure, by changing the Signs + or — of thoſe Terms, where we meet with r or ρ, or making them infinite, according as the reſpective Surfaces are *concave* or *plane*. Thus in the caſe of a *plano-convex*, r being infinite, the Difference ariſing upon turning the *Lens*,

becomes $\frac{Rrt}{Ir} = \frac{R}{I}t$; or in Glaſs $\frac{2}{3}t$, in Water $\frac{3}{4}t$, and $\frac{2}{5}t$ in Diamond. Which ſhews that the *Focal* Diſtance is greater by $\frac{2}{3}t$ when the *plane* ſide of a *plano-Convex* of Glaſs is turn'd towards a *vaſtly diſtant* Object, then when the *convex* ſide is turn'd to it. After the ſame manner, the negative *Focal* Diſtance of a *plane Concave* will be greater by $\frac{2}{3}t$, when the *plane* ſide is turn'd towards the *vaſtly diſtant* Object, than when the *concave* ſide is turn'd to it. In *double-concaves*,

caves, where the *Focus* is always negative, the Difference is $\frac{\pm 2\rho t \mp 2rt}{-3r - t - 3\rho}$, according as ρ is greater or less than r. In *Menisci*, the *Focal* Distance, whether affirmative or negative, being always greatest when the *concave* Surface is turn'd towards a vastly distant Object, the Difference is $\frac{-2rt - 2\varrho t}{-3r - t + 3\rho}$ when the *Foci* upon turning either side fall both one way, and are either both affirmative or negative: But if t be so considerable as to be greater than 3ϱ, the *Focal* Distance upon turning the *concave* side $\frac{-6r\varrho - 2\rho t}{-3r - t + 3\rho}$ is affirmative, and the *Focal* Distance upon turning the *convex* side $\frac{-6\varrho r + 2rt}{3\varrho - t - 3r}$ negative, and consequently their Difference is $\frac{2rt - 12r\rho - 2\varrho t}{-3r - t + 3\varrho}$: And if t were equal to 3ϱ, the *Focus* in this case upon turning the *convex* side to a *vastly distant* Object, falling exactly upon the *vertex* of the second Surface of the *Lens*, and consequently the *Focal* Distance being equal to nothing, the Difference will be the same with the *Focal* Distance upon turning the *concave* side, namely, $\frac{-6r\varrho - 6\varrho\varrho}{-3r}$. After

the like manner, may be found from the general Rule, the Difference which would arise upon turning the different Surface of any sort of *Lens* towards other Rays besides *parallel*, whether *diverging* or *converging*; but the *Canons* for these Cases consist of so many Terms, and are of so little use, that they are not worth having.

There are three farther *Uses* to be made of the general Rule above deliver'd; The first is, from the *Lens* and *Focus*, where an Object is represented, being given, to determine the *Distance* of the Object from the *Lens*; or the *Lens* by which we wou'd form the Image of any Object, and the *Focus* where we would have it form'd, being given, to determine the Distance at which the Object should be placed before the *Lens*, that it may be represented in the given *Focus*. That is, in the Equation $y = \frac{pdr\rho}{dr + d\rho - pr\rho}$, or $dry + d\rho y - pr\rho y = pdr\rho$, r, ρ, p, and y being given, 'tis required to find d, and consequently we shall have $d = \frac{pr\rho y}{ry + \rho y - pr\rho}$; where 'tis plain, that if r and ρ be so great in respect to y, that $pr\rho$ exceeds $ry + \rho y$, d will be negative; and the Object cannot be represented in the Circumstances requir'd, unless by means of another *Lens*, we first make

make the Rays coming from the Object, *diverging* converge to a Point behind the first Surface of the *Lens* given, at the Distance of $\frac{pr\rho y}{pr\rho - ry - \rho y}$: And if $pr\rho$ is equal to $ry + \rho y$, d will be infinite. Suppose the given *Lens* a *double convex* of Glass, and made of two Segments of equal Spheres, but of a thickness not considerable, and it is required to find at what Distance from the *Lens* a lucid Body should be placed, in order to have its Beams parallel after their Emersion from the *Lens*, and consequently its Light thrown upon Objects *vastly distant*, which may be thereby illuminated : In this case y being infinite, and r equal to ρ, and p equal to 2, we shall have the Distance requir'd $d = r$. But if t be consiberable, we must find d from the Rule which takes in the thickness of the *Lens*, which gives us the exact value of $d = \frac{4r\rho t + 6r\rho y - 2rty}{3ry - ty + 3\rho y - 6r\rho + 2\rho t}$: As, if for Example, the *double-convex* just mention'd, were an entire *Sphere* of Glass, and the same thing requir'd as before, y being, as we have already observ'd, infinite, and r equal to ρ, and moreover t equal to $2r$; this last Rule gives $d = \frac{6rry - 4rry}{3ry - 2ry + 3ry} = \frac{1}{2}r$, whereas by the former which neglects

 the

the thickneſs, we have the Diſtance requi-red twice as great, or $d = r$; a Difference very conſiderable, if the Spheres be of any bigneſs. So then, a lucid Body placed at the Diſtance of half the Radius from a Sphere of Glaſs, or at the Diſtance of the whole Radius from a *double-convex* of equal Spheres, whoſe thickneſs is inconſiderable, will illuminate Objects vaſtly diſtant. If the given *Lens* were a *Hemiſphere* of Glaſs, and the ſame thing ſtill requir'd; if the *convex* Surface be firſt, both y and ρ being in this caſe infinite, and t equal to r, the Diſtance of the lucid Body will be $d = \frac{6r\rho y}{3\rho y} = 2r$, but if the *plane* ſide be next the lucid Body, y and r being infinite, we have $d = \frac{6r\rho y - 2rty}{3ry} = \frac{4}{3}\rho$ or $\frac{4}{3}r$; there being as has been ſhewn before a Difference of $\frac{2}{3}t$ in the *Focal* Diſtance of a *plano-convex* expos'd to *parallel* Rays, occaſion'd by turning the different ſides of the *Lens*: If t had been neglected, we ſhould have had $d = 2r$ in both caſes. If we have an Object repreſented by a *double-concave* of Glaſs of equal Spheres, at a negative *Focus* the Diſtance of the Radius from the *Lens*, and it were requir'd to find the Diſtance of the Object, y, r, and ϱ being all negative and equal, and t inconſiderable, we ſhall have

$d =$

$d = \frac{-2rrr}{2rr - 2rr}$ infinite, and conſequently the *Object* is *vaſtly diſtant*. The ſame thing may be done for all other Caſes whatever, only remembring to make the proper Alterations according as r, ϱ, or y are negative or infinite, and t conſiderable or inconſiderable.

The ſecond *Uſe* is, from one Surface (either the firſt or ſecond) of a *Lens* being given already form'd, to find what degree of *convexity* the other Surface muſt have, in order to repreſent a given Object at a given *Focus*. That is, in the Equation before uſed $dry + d\varrho y - pr\varrho y = pdr\varrho$, d, y, p and r being given, to find ϱ, or ϱ being given to find r: Whence we have $\varrho = \frac{dry}{pdr - dy + pry}$, and $r = \frac{d\varrho y}{pd\varrho - dy + p\varrho y}$; which will ſerve for any other *Lens*'s beſides *double-convex*, and any other Rays beſides *diverging* ones, by making ſuch alterations as has been already directed. If the firſt Surface of a Glaſs *Lens* were *plane*, and it were requir'd to find what Degree of Convexity the ſecond Surface muſt have, in order to repreſent an Object at a *Focus* juſt as far diſtant from the *Lens* as the Object it ſelf: In this caſe d is equal to y, and r infinite, and conſequently $\varrho = \frac{dry}{2pry} = \frac{1}{4}d$, ſo that the ſecond Surface of the *Lens* muſt

be

be made of a Segment of a Sphere, whose Radius is equal to $\frac{1}{4}$ of the Distance of the Object. If the Object to be represented at a given *Focus*, be *vastly distant*, *d* being in this case infinite, the Rule is abridg'd to $\rho = \frac{ry}{pr - y}$; whence 'tis plain, that if in Glass *y* is greater than $2r$, or the given *Focus* be at a greater Distance from the *Lens* than twice the Radius of the given Surface, ρ will be negative, and the second Surface must be made *concave*; and if *y* be equal to $2r$, ρ is infinite, and the second Surface must be *plane*. If the thickness of the *Lens* be so great, that it ought to be consider'd, we must find ρ from the general Rule.

The third *Use* is, from the *Lens*, *Distance* of the Object, and *Focus* being given to determine the *Ratio of Refraction*. That is, in the Equation before used $dry + d\rho y - pr\rho y = pdr\rho$, d, r, ρ and y being given, to find p, which gives us $p = \frac{dry + d\rho y}{dr\rho + r\rho y}$: For p being found, the value of $p = \frac{\mathrm{I}}{\mathrm{I} - \mathrm{R}}$ gives the *Ratio* I to R as $\mathrm{I} + p$ to p. If a *double-convex Lens*, made of two Segments of the same Sphere, represents, or is required to represent a vastly distant Object, at a *Focus* the Distance $\frac{1}{2}r$, from the *Lens*; *d* being in this case infinite, and

and r equal to ρ, and y equal to $\frac{1}{2}r$, we shall have $p = \frac{3drr}{drr} = 3$, and consequently the *Ratio* I to R as 4 to 3; whence the *Lens* is made, or ought to be made of the *medium* of Water. If the *Focal* Distance were $\frac{1}{3}r$, we should have $p = \frac{2}{3}$, or I to R as 5 to 2, and consequently the *Lens* would be *Diamond.* If the *Focal* Distance were $4r$, then we have $p = 8$, or I to R as 9 to 8, and the *Lens* is *Glass*, and the *ambient medium Water.* But if we are very curious in determining the *Ratio* of *Refraction*, it is done more exactly when the *Lens* is form'd into an *Hemisphere*, or a *plano-Convex*, and receiving the Rays of the Sun upon its *plane* side, collects them in a Point at some Distance behind, which must be measured with great niceness; because in this case, our neglecting t occasions no Error at all. In this case, if the *Focal* Distance is equal to thrice the Radius of the Sphere, d and r being infinite, and y equal to 3ρ, 'twill be $p = 3$, or I to R, as 4 to 3; if y is equal to 2ρ, I is to R as 3 to 2; and if y is equal to ρ, then I is to R as 2 to 1.

V. At Prop. XX. Probl. XIII.

WHich is, *To find the Distance at which an Object should be placed from a given Lens, so as that the Image form'd by the Lens may bear a given Proportion to the Object.*

The Author has given the *Construction*, but omitted the *Demonstration*; leaving the process of the Calculation which points out that Construction, as a tryal of Skill to the diligent Reader. But because my design in publishing the Book is to make it entirely easy; for fear it may prove too difficult or discouraging a Task to some who are either not skilful enough, or perhaps too lazy to go through with it, I have subjoin'd the following *Solution.*

[*Plate III. Fig.* 6, 7.] Let the given *Lens* BD be a *double-convex*, and call AB the Radius of the first Surface *a*, CD the Radius of the second Surface *b*, BD the thickness of the *Lens* C, the *Proportion* of the *homologous* Lines of the Object and Image as *r* to *h*, and

and EB the Diſtance required z, at which the Object is to be placed before the *Lens* B, which we ſuppoſe made of Glaſs and the ambient *medium* Air. 'Tis plain there are two different Values of EB (z), according as the *Focus* F is affirmative, or beyond the *Lens*, as at *Fig.* 6. Or negative, and on the ſame ſide with the Object, as at *Fig.* 7. Both which Caſes ſhall be reſpectively conſider'd, and included in the Demonſtration.

By *Corol. Prop. XVIII.* the homologous Lines of the Object and Image are to another as their reſpective Diſtances from the *Lens*; wherefore r is to b, as EB (z) the Diſtance of the Object required to DF the *Focal* Diſtance of the deſir'd Image, which is conſequently $\frac{bz}{r}$. But we ſhall have another value of this *Focal* Diſtance from *Prop. XV.* For if we look upon f as the *Focus* of Rays ſent diverging from the Object at E, after their Refraction at the firſt Surface of the *Lens*, and F their *Focus* after both Refractions; and call the firſt *Focal* Diſtance Bf x, and the ſecond DF y. Before we can find the *Focal* Diſtance DF, which determines the place of the *Image*, we muſt firſt find Bf. Now to find Bf, by *Prop. XV.* $\frac{EA}{3}\left(\frac{z+a}{3}\right)$: AB (a) :: Ef ($\pm z+y$):

Bf (y); whence $zy + ay = \pm 3az + 3ay$, and Bf (y) $= \frac{\pm 3az}{z - 2a}$. Having found Bf, the same Proposition gives us DF, for $\frac{fC}{2}$ $\left(\frac{\frac{\pm 3az}{z-2a} \mp c \pm b}{2} = \frac{\pm 3az \mp cz \pm bz \mp 2ac \mp 2ab}{2z - 4a}\right)$: CD (b) : : fF $\left(\frac{3az}{z-2a} - c \mp x = \frac{3az - cz \mp xz + 2ac \pm 2ax}{z - 2a}\right)$: DF (x); whence $\pm 3azx \pm czx \pm bzx \mp 2acx \mp 2abx = 6abz - 2bcz \mp 2bzx + 4abc \pm 4abx$, and DF $= \frac{6abz - 2bcz + 4abc}{\pm 3az \mp cz \pm 3bz \pm 2ac \mp 6ab}$. Therefore comparing the two values of DF together, we have this Equation $\frac{bz}{r} = \frac{6abz - 2bcz + 4abc}{\pm 3az \mp cz \pm 3bz \pm 2ac \mp 6ab}$, or $\pm 3abz^2 \mp bcz^2 \pm 3bz^2 \pm 2abcz \mp 6ab\,z = 6abrz - 2bcrz + 4abcr$. From whence we have $ZZ = \frac{6abZ - 2aCZ \pm 6arZ \mp 2rCZ \pm 4arC}{3a - C + 3b}$; And if the thickness of the *Lens* be neglected as inconsiderable, all the Terms where

C

C occurs vanishing, we have $zz = \frac{6abz \pm 6arz}{3a+3b}$, and consequently EB ($z$) $= \frac{2ab \pm 2ar}{a+b}$. *Q. E. I.*

[*Plat.* II. *Fig.* 22.] If then we would *construct* this Equation, we have this Proportion given us for that purpose, $a+b : b \pm r :: 2a : z$. Taking therefore A B equal to a, and B C in the same right Line equal to b; from C draw at pleasure the indefinite right Line C M, upon which cut off C D equal to C B (b), and from D on either side take D M, or Dm such, that it may bear the same *Proportion* to D C, which the *Homologous* Lines of the Object do to those of the Image. Join A M or Am, to which thro' B draw B E or Be parallel; and twice M E or twice me is the *Distance required* in the *Problem*. For A C ($a+b$): M C or mC ($b \pm r$) :: 2AB ($2a$) 2 ME or 2 me (z). And consequently the *Construction* gives the true value of z, as before found. *Q. E. D.*

The same *Problem of magnifying or diminishing a given Object by a given Lens, in any assign'd proportion*, may be solved from the Equation above given, for finding the *Foci* of all sort of *Lens*'s, which if express'd in the Characteristicks in present

use, is $r = \frac{pabz}{az+bz-pab}$. For supposing as before r to b expresses the proportion which the *Homologous* Lines of the Object are required to bear to those of the Image, and 'tis desir'd to find z the Distance of the Object from the *Lens*, which is necessary to perform the Conditions required: By *Corol. Prop. XVIII.* we have another value of the *Focal* Distance $x = \frac{bz}{r}$. Whence comparing both together we have $\frac{bz}{r} = \frac{pabz}{az+bz-pab}$, and consequently if the *Focus* is to be affirmative $z = \frac{pab+par}{a+b}$, but if pab be greater than $az+bz$, then the *Focus* is negative, or on the same side with the Object, and this negative *Focal* Distance is $\frac{pabz}{-az-bz+pab}$, and consequently $z = \frac{pab-par}{a+b}$; That is, if the *Lens* be Glass, the following Equation includes both cases, whether the Image is to be represented on the contrary or same side with the Object, $z = \frac{2ab \pm 2ar}{a+b}$. Where it is to be observ'd, that if the Image is represented on the contrary side by a *double-convex Lens*, or at an affirmative *Focus*, it may be made either equal

equal to, greater or leſs than the Object in what proportion we pleaſe; but if it is repreſented on the ſame ſide, or at a negative *Focus*, r muſt always be leſs than b, and conſequently the Image may be ſhewn larger than the Object in all the degrees imaginable, but never leſs, nor equal; for when b is equal to r, and the *Focus* negative, z is $= o$, and when b is leſs than r, z is negative and impoſſible.

Nor is this Solution confin'd to the caſe of *double-convexes* only, tho' made for *Lens*'s of that Figure, but will with proper Alterations extend equally to *Lens*'s of all other ſorts whatever; only obſerving to change the Signs $+$ or $-$ with which the Radius of a *concave* Surface is affected, or making the Terms infinite where the Radius of a *plane* Surface occurs; becauſe it has been ſhewn before, that the Radius of a *concave* Surface bears a contrary Sign to that of a *convex*, and the Radius of a *plane* Surface is infinite. If then the *Lens* be a *concavo-Convex* or *Meniſcus* of Glaſs, and the firſt Surface concave, the Rule for both caſes, whether the *Focus* is to be affirmative or negative, is $z = \frac{-2ab \mp 2ar}{-a+b}$; and if the ſecond Surface be that which is *concave*, then for the affirmative and negative *Foci* reſpectively, the Rule becomes $z = \frac{-2ab \pm 2ar}{a-b}$. Where it is to be obſerv'd, that

that in the firſt caſe if the *Focus* be affirmative, *a* muſt be greater than *b*, or elſe *z* will be negative, and the Problem impoſſible; and if the *Focus* be negative, and *b* greater than *r*, then muſt *a* be ſtill greater than *b*, but if *b* be leſs than *r*, then *a* muſt be leſs than *b*; or elſe the Problem will be impoſſible: And in the laſt caſe, if the *Focus* be affirmative, and *b* greater than *r*, then *b* muſt be greater than *a*, and *vice verſa*; and if the *Focus* be negative, *b* muſt ſtill be greater than *a*. Which ſhews that in the caſe of a *Meniſcus* turn'd on the *concave* ſide towards the Object, the Image can never be repreſented at all on the oppoſite ſide, unleſs the *concave* Surface be a Segment of a larger Sphere than the *convex*, and then it may be ſhewn in what Proportion to the Object we pleaſe; and if it is to be repreſented on the ſame ſide with the Object, and magnified, the Radius of *concavity* muſt be ſtill larger than that of *convexity*, and *vice verſa*, if the Image is to appear diminiſhed. And in like manner may be underſtood what will happen upon turning the convex ſide of the *Meniſcus* towards the Object. If the *Lens* be a *double-concave* of Glaſs, the *Focus* being in this caſe always negative, we have but one value of z, which is $\frac{2ab - 2ar}{-a - b}$, affirmative only when r is greater than b; which ſhews that

a

a *double-concave* can only diminiſh. If the *Lens* be a *plano-Convex* of Glaſs, 'twill be $z = 2b \pm 2r$; which ſhews that in this caſe the Object may either be magnified or diminiſhed, if the *Focus* be affirmative, but only magnified if it be negative. It muſt be noted, that if the ſecond Surface, whoſe Radius is b were ſuppos'd plane, we ſhou'd have $z = 2a$, becauſe b not only ſtands for the Radius of that Surface, but alſo expreſſes the *ratio* of the Image to the Object, which is conſequently in this caſe infinite, and the Image *vaſtly diſtant*. If the *Lens* be a *plano-Concave* of Glaſs, the *Focus* being always negative, we have only one value of z, which is $z = -2b + 2r$, which ſhews that a *plano-Concave* can only diminiſh.

It may be remark'd, That thoſe Caſes which make the Diſtance z negative, and the Problem impoſſible for the given *Lens*, may by means of another *Lens* be made practicable: If we firſt receive the Rays of the Object upon this ſecond *Lens*, and before they are collected at the *Focus* throw them upon the given *Lens*, in ſuch a manner as to make them fall *converging* to a Point behind the firſt Surface of the given *Lens*, at the Diſtance of the negative value of z.

If

If the *Lens* were a *double-convex* of *Water*, p being $= \frac{R}{1 - R}$, 'tis $z = \frac{3ab \pm 3ar}{a + b}$; if of *Diamond* $z = \frac{\frac{2}{3}ab \pm \frac{2}{3}ar}{a + b}$; if of *Glaſs* in an ambient *Medium* of *Water* $z = \frac{8ab \pm 8ar}{a + b}$; if of *Diamond* in an ambient *Medium* of *Water* $z = \frac{\frac{8}{7}ab \pm \frac{8}{7}ar}{a + b}$, and conformably in all other Caſes.

If the Image be deſir'd juſt as great as the Object, then for a *double-convex* of Glaſs, r being equal to b, the Rule gives the Diſtance required $z = 2b \pm 2a$; where 'tis plain the Problem is always poſſible when the *Focus* is affirmative, but if the *Focus* is negative, 'tis only poſſible when b is greater than a. If the *Lens* were equally *convex*, b being in this caſe equal to a, there will only be one value of z, and that for an affirmative *Focus* $z = 4a$: So that if the Object be placed at four times the Diſtance of the Radius from the *Lens*, the Image form'd at its *Focus* will be juſt as great as the Object. If the *Homologous* Lines of the Image were deſir'd twice as large as thoſe of the Object, r being in this caſe equal to $\frac{1}{2}b$, for a *double-convex* of Glaſs the Rule gives $z = 2b \pm a$, and if the *Lens* be equally

convex

convex $z = 3a$ or a according as the *Focus* is affirmative or negative, ſo that an Object placed at the diſtance either of thrice or only once the Radius from the *Lens*, is repreſented at a *Focus*, either affirmative or negative, twice as large every way as the Object, or the Image in its whole content will be four times as large as the Object. If the *Homologous* Lines of the Image were deſired twice as ſmall as thoſe of the Object, r being in this caſe equal to $2b$, the Rule gives for an equally convex *Lens* only one value of the Diſtance $z = 6a$, a *double-convex* being only capable of repreſenting an Object diminiſh'd, when the *Focus* is affirmative. If the Image be deſir'd an hundred times larger than the Object, or its *Homologous* Lines ten times as large, r being in this caſe equal $\frac{1}{10}b$, if the *Lens* be equally *convex*, the Rule gives the Diſtance $z = 2\frac{1}{5}a$, or $\frac{9}{5}a$, according as the *Focus* is affirmative or negative. And in general if m to n expreſs the Proportion which r bears to b, the Rule laid down at firſt $\frac{pab \pm par}{a+b}$, becomes $pb \pm p\frac{m}{n}a$, and if the *Convexity* be equal, 'tis $z = pa \pm p\frac{m}{n}a$. From all which it appears that with a *Lens* equally Convex on both ſides, in order to magnify a given *Object*, the Diſtance is always ſomething grea-

ter than pa, when the *Focus* is affirmative, and always leſs than pa when the *Focus* is negative, unleſs the *Ratio* of the Image to the Object be infinitely great, and then n being infinite, 'tis $z = pa$. That is, in Glaſs the Diſtance of the Object muſt always exceed twice the Radius, if the *Focus* is to be affirmative, or fall ſhort of it if negative, and be equal to it when the Image is to be infinitely great, or vaſtly diſtant. And in order to diminiſh a given Object $\frac{m}{n}$ in this caſe exceeding *Unity*, the Diſtance, which has but one Caſe here, is always greater than $2pa$; and the more z exceeds $2pa$ the more the Object is diminiſhed, and *vice verſa*, till z becoming equal to $2pa$, the Object and Image are likewiſe equal.

All this is eaſily obſerv'd in that common Experiment of the *Camera Obſcura*: Where the Rays propagated from External Objects are received by a *Lens*, and tranſmitted into the Room, and do there paint upon a white Sheet placed at the *Focal* Diſtance of thoſe Rays from the *Lens*, the Images of their reſpective Objects, in Colours ſcarce leſs lively than thoſe of the Objects themſelves. And hence this Problem of magnifying or diminiſhing a given Object, may be of great uſe in *painting*; for by admitting the Image of any Object by means of a *Lens* into a dark Chamber, in what *Ratio*

to the Life we pleaſe, 'tis eaſy to hit the proportion of every part with great exactneſs, which is otherwiſe but ſeldom done; eſpecially in ſuch Pieces where the Figures are either much greater or much leſs than the Life. 'Tis true, theſe Images that are repreſented in the *dark Chamber* by a ſingle *Lens* appear inverted; but may be made erect by uſing a ſecond *Lens* after the following manner. Place the *Object* at ſuch a Diſtance from the firſt *Lens* in the Window, that the *Image* form'd by that, may be juſt as big as the Life; then beyond the place of this *Image* fix the given *Lens* at the Diſtance requir'd z; this will form an *Image* of the former *Image* which ſhall be in the *Ratio* aſſign'd, and alſo erect, but ſomething leſs lively. What has been above deliver'd is likewiſe of ſervice in the Conſtruction of the *Magick Lanthorn*, and other Optical Machines, where the *Images* of any *Objects* are to be repreſented monſtrouſly larger or leſs than the Life.

VI. At Prop. XXIV. Probl. XVII.

WHich is, *To make a concave* Speculum *of Glaſs of a given Thickneſs; the Radius of whoſe concavity is alſo given, in ſuch a manner, that parallel Rays reflected from the firſt Surface of the* Speculum *may meet in the ſame Point of its Axis, with thoſe that paſs refracted into the Speculum, and are reflected from the ſecond Surface, and again refracted at their Emerſion from the Speculum*, Or the firſt Surface of a *concave Speculum* being already form'd, to determine of what Sphere 'tis neceſſary to take a Segment to form the ſecond Surface in ſuch manner, that an *Object vaſtly diſtant* may be repreſented by Reflection from both Surfaces in one and the ſame Place, or that the two Images may be united, and conſequently be made more lively. The Author has given two ſeveral *Conſtructions* of the *Problem*; the firſt exact, and the

the other only near the Truth, but more expeditious: But for Reaſons already mention'd he has omitted their *Demonſtration*, which is as follows.

[*Plat. III. Fig.* 8.] Let AB be the given Radius of the firſt Surface, and BD the given Thickneſs of the *Speculum*, and CD the Radius required of the hindmoſt Surface neceſſary to perform the Conditions of the *Problem*. Suppoſe the Point f in fD, the Axis of the *Speculum* to be the *Focus* of the *parallel Rays*, after *Refraction* at their entrance into the firſt Surface, φ their *Focus* after *Reflection* from the ſecond Surface, and F their *Focus* after *Refraction* again at their Emerſion from the *Speculum*; 'tis required that the Point F ſhould be the ſame with the *Focus* of the ſame *parallel Rays* after their *Reflection* from the firſt Surface. Call the given Radius ABa, the given Thickneſs BDc, the firſt *Focal* Diſtance B$f$$v$, the ſecond D$\varphi$$x$, the laſt *Focal* Diſtance BFy, and the required Radius of the hindmoſt Surface CDz. We muſt find theſe *Focal* Diſtances one after another, in order to determine the laſt BF, which muſt be equal to the *Focal* Diſtance of *parallel Rays* reflected from a *concave Speculum*. For the firſt then Bf, by *Prop. XIV.* Bf (v) : fA $(v-a)$: : I : R : : (in Glaſs) 3 : 2; whence Bf $(v) = 3a$. And for the ſecond

second Dφ, by *Prop. IV.* Dφ (x) : φC $(z-x)$ :: Df $(3a+c)$: fC $(3a+c-z)$; whence $3ax+cx-zx=3az+cz-3ax-cx$, and consequently Dφ $(x) = \frac{3az+cz}{6a+2c-z}$.

Lastly, To determine the *Focal* Distance BF after both *Refractions* at the first Surface, and the *Reflection* from the last, by *Proposit. XV.* the Emersion being out of Glass into Air, $\frac{\varphi A}{2} \left(\frac{a+c-\frac{3az-cz}{6a+2c-z}}{2} = \frac{6aa+8ac-4az+2cc-2cz}{12a+4c-2z} \right)$: AB (a) :: φF $\left(\frac{3az+cz}{6a+2c-z} - y - c = \frac{3az+2cz-6ay-2cy+zy-6ac-2cc}{6a+2c-z} \right)$: BF (y); whence $6aay+8acy-4azy+2ccy-2czy=6aaz+4acz-12aay-4acy+2azy-12aac-4acc$, and consequently BF $(y) = \frac{3aaz+2acz-6aac-2acc}{9aa+6ac-3az+cc-cz}$.

Now this must be equal to the *Focal* Distance of *parallel Rays* reflected from a *concave Speculum*, which by *Prop. III.* is just half the Radius, whence we have another value of BF $(y) = \frac{a}{2}$.

And

And comparing both together we have $\frac{3aaz + 2acz - 6aac - 2acc}{9aa + 6ac - 3az + cc - cz} = \frac{a}{2}$, or $9az + 5cz - 18ac - 5cc = 9aa$, and consequently the Radius required $CD (z) = \frac{9aa + 18ac + 5cc}{9a + 5c}$. *Q. E. I.*

Fig. 3. If then we would *Construct* this value of $CD(z)$, we have the following Proportion given us for that purpose $9a + 5c : \sqrt{9aa + 18ac + 5cc} :: \sqrt{9aa + 18ac + 5cc} : z$. Wherefore making the Rectangular Triangle MLK in such manner, that LM shall be equal to $9a + 5c$, and LK equal to $\sqrt{9aa + 18ac + 5cc}$, and then drawing from K the Perpendicular SK, the Right Line LS is the Radius required. For by *Element. VI.* 8. $LM\ (9a + 5c) : LK\ (\sqrt{9aa + 18ac + 5cc}) :: LK\ (\sqrt{9aa + 18ac + 5cc}) : LS\ (z)$. And consequently the *Construction* gives the true value of $CD\ (z)$ as before found. *Q.E.D.*

Fig. 4. The Radius required is also capable of another *Construction*, for making an actual *Division* of $9aa + 18ac + 5cc$ by $9a + 5c$, the *Quotient* is $a + c + \frac{4ac}{9a + 5c}$; And if $AB\ (a)$ be sufficiently great in respect to $BD\ (c)$, the Term $5c$ in the *Denominator*

nominator of the *Fraction* may be neglected, and then it becomes $z = a + c + \frac{4}{9}c$, whose excess above the Truth is not at all sensible. Wherefore taking DO equal to $\frac{4}{9}$ of BD(c), and making AC equal to DO, CD $(a+c+\frac{4}{9}c)$ is very nearly the true value of the *Radius required.* *Q. E. D.*

To shew how near this last value of $z=a+c+\frac{4}{9}c$ is to the Truth, if we suppose the Thickness of the *Speculum* to be $\frac{1}{5}$ of the given Radius of the first Surface, which is very considerable: In this case the exact value of $z=a+c+\dfrac{4ac}{9a+5c}$ is but $\dfrac{2}{225}a$ or nearly $\frac{1}{112}a$ less than its value found by neglecting $5c$ in the other Equation $z=a+c+\frac{4}{9}c$, a Difference not all considerable in *Physical* Matters.

If c be supposed equal to $\frac{1}{3}a$, which is still a far greater Supposition, even in this Case, the value of the Radius z taken from the last Equation $z=a+c+\frac{4}{9}c$, is but $\frac{15}{628}a$ or very nearly $\frac{1}{42}a$ greater than the Truth, which is an excess not very sensible, unless a be extremely great. But if the Thickness be greater than in this last Supposition, it will be convenient to take the value of the Radius required from the exact Equation $z=a+c+\dfrac{4ac}{9a+5c}$. As, if c be equal to $\frac{1}{2}a$, the Radius required is $1\frac{11}{46}a$, which is

is about $\frac{1}{21}a$ leſs than what it would be if $5c$ were neglected. If c be equal to a, the Radius required is $2\frac{2}{7}a$, which is about $\frac{1}{6}a$ leſs than it would be if $5c$ were neglected. If the Thickneſs be ſo conſiderable as to be equal to thrice the given Radius of the firſt Surface, then the Radius required is $4\frac{1}{2}a$.

Hence likewiſe if the Radius of the laſt Surface be given, together with the Thickneſs of the *Speculum*, we may find the Radius of *concavity* neceſſary to unite the two Images of a vaſtly diſtant Object made by *Reflection* from both Surfaces. For if c be not very great, we ſhall have $a = z - c - \frac{4}{9}c$, as near the Truth as need be required in Practice: For if we had the exact value of a, we cou'd not in practice grind the *Speculum* to the due Concavity, even ſo near as the value juſt now given. If c be conſiderable, the value of a muſt be found from an Equation of an higher Degree $9aa - 9az + 18ac = 5cz - 5cc$; which if it be contracted, by putting p for $2c - z$, will give $a = \mp \frac{1}{2}p \pm \sqrt{\frac{5cz - 5cc}{9} + \frac{1}{4}pp}$, the Sign of $\frac{1}{2}p$ being either $-$ or $+$ according as $2c$ is greater or leſs than z. After the ſame manner, having the Radius of both Surfaces given, we may find what

Thickneſs of the *Speculum* is neceſſary to unite the two Images of a *vaſtly diſtant Object*, form'd by *Reflection* from both Surfaces, by means of the following Equation, $5cc + 18ac - 5cz = 9az - 9aa$, which if it be contracted, by putting q for $\overline{3\tfrac{3}{5}a - z}$, will give $c = \mp \tfrac{1}{2}q \pm \sqrt{\tfrac{9az - 9aa}{5} + \tfrac{1}{4}qq}$.

VII. A more particular Account of MICROSCOPES and TELESCOPES, from Mr. *Huygens*.

PROP. I.

TO *explain the Effects and Uses of* single Microscopes, *and the manner of making little* Spheres *and* Lens's.

[*Fig.* 9, 10.] Let N be the *Lens* QRQ, the Object at its *Focus* R, O the Eye very near the *Lens*. The Rays coming from R will after their Refraction fall parallel upon the Eye, and consequently make distinct Vision. For the Fabrick of the Eye, having its *Focal* Distance just at the Bottom of it upon the *Retina*, requires that the Rays from each single Point should fall nearly parallel in order to be there collected: that is, that the Basis of each Cone of Rays flowing from every Point of any Object, which Basis is the Pupil of the Eye, should bear so small a proportion to the Length of the Cone, as

that thoſe Cones may be look'd upon as little Cylinders. (The Diſtance indeed requiſite for diſtinct Viſion is not limited to a point, but is indulged in larger Bounds; becauſe Nature has furniſh'd us with the power of contracting the Pupil as the Object comes nearer, and ſo diminiſhing the Baſis of each Cone in proportion, and conſequently of preſerving diſtinct Viſion, but this is only to a certain and that no very great Degree.) But the Object QRQ will appear in the ſame magnitude, as if the *Lens* N were removed, and a Plate with a ſmall hole in it ſubſtituted in its place, namely under the Angle QAQ. So that all the interpoſed *Lens* does in this caſe, is only making diſtinct Viſion which would without the *Lens* be confuſed. But ſince at the diſtance ſuppoſe of 8 Inches from the Object, a naked Eye has then diſtinct Viſion; the apparent Image may be ſaid to be ſo much magnify'd as thoſe 8 Inches exceed the little Space NR, or the *Focal* Diſtance of the *Lens* N: which if it be equal to $\frac{1}{5}$ of an Inch, the appearance of the Image ſeen diſtinctly by the *Microſcope*, is to that ſeen diſtinctly by the naked Eye as 40 to 1. Therefore the leſs the *Focal* Diſtance of the little *Lens* N is, the greater will its Effects be in dilating the Image of a ſmall Object; tho' there are ſome Inconveniences (to be mention'd afterwards) which here offer

themſelves,

themſelves, and forbid our going beyond ſome certain Limits. And the ſame thing happens to little *Spheres*, which may be us'd for *Lens*'s, and might otherwiſe be made as little as we pleaſe. But theſe ſmall *Spheres* are inferiour to little *Lens*'s upon this Account, that for the ſame degree of magnifying, if both be made of Glaſs, the *Lens*'s are three times more diſtant from the Object than the *Spheres*; and by that means leave a ſufficient Space for the lateral Light to enter, and make the Colours of the Object viſible; whereas otherwiſe we are forced to turn the *Microſcope* directly againſt the Light, and can only than diſcern diſtinctly ſuch Objects as by their thinneſs are pellucid.

The Effects of a little *Sphere*, and what has been ſaid concerning the three times leſs Diſtance is thus demonſtrated. *Fig*. 11. 9.

Let there be a Glaſs *Sphere* whoſe Centre is K, and its Axis AB, in which produced on both ſides the Eye is plac'd at D, and the Object at C, each of the Diſtances AD, BC being taken equal to half the Radius AK; And conſequently the Point C is the *Focus* where Rays falling parallel to the Axis AB, upon the *Sphere* at AH are after emerſion collected, as is ſhewn at ADDEND. IV. Wherefore an Object plac'd at C will ſend Rays upon the *Sphere*, which will after Refraction be receiv'd parallel by the Eye, and

conſequently

consequently make distinct Vision. But by *Prop. XIV.* if we take the Point F such, that FA may be equal to the Radius AK, the Point OF is the *Focus* towards which parallel Rays after Refraction at the first Surface BG do tend in their passage thro' the *Sphere*, and from which they are diverted after Refraction at their Emersion and collected at D. Make GE parallel to the Axis, and comprehending the Portion of the Object CE, and draw the right Line ED. The Ray then EG being refracted at G, proceeds according to GF, and being again refracted at H, goes on to meet the Eye at D. Wherefore the Line CE is seen under the Angle ADH, which would appear to the naked Eye under the Angle CDE, which I say is but the half of the former.

For because AF is double of AD, the Angle ADH is double of AFH. But DF is parallel to EG, because GE is both parallel to FD, and to be look'd upon as equal to it or to the right Line BC; because CE is to be a Line very small with respect to the Diameter of the *Sphere*. Therefore the Angle ADH is also double of the Angle CDE, and consequently equal to the Angle CKE. From whence it is plain that to the Eye placed at D, the Line CE will appear under the same Angle, in which it wou'd appear to the naked Eye seeing from the Point K.

Whence

Whence if the Diameter of the little *Sphere* AB were $\frac{1}{12}$ of an Inch, we should have KC $= \frac{1}{16}$ of an Inch, which is to the Distance of 8 Inches in the proportion of 1 to 128; so that the increas'd Magnitude of the Object wou'd be as 128 to 1, which is indeed very considerable. But if NR the *Focal* Distance of the *Lens* be equal to the right Line KC, we have shewn that by this means the Object RQ would be seen in the same Magnitude as if the Eye were plac'd at N without the *Lens*; Nor in the using this *Lens* will the apparent Magnitude be any ways changed, in whatsoever part of the Axis RN produc'd the Eye be plac'd. Therefore 'tis plain the same degree of Magnifying, and the same Effect every way is perform'd equally by the *Lens* N and the little *Sphere* AB. And it is moreover manifest that the Distance RN, being taken equal to KC, is equal to thrice BC, *Q. E. D.*

We are next to explain the manner how little Spheres *and* Lens'*s may be prepar'd and fitted for Use.*

The less *Spheres* are to be, the easier they are prepar'd after the following manner. Take the smallest Fragments of Glass, and hold them to the lower part of the Flame of a Candle where the bluish Colour is discernible, that they may grow red hot; and then if they be taken up by the finest Steel-Wire that can be got, and dexterously turn'd, they will be form'd into Globules,

Globules, which are large enough if equal to a Grain of Muſtard-Seed. Out of ſeveral thus prepar'd, you will find ſome very good; which may be tried by including them in a Braſs-plate, and is thus done. Take a Plate of the thinneſt Braſs the breadth of a Finger, and twice as much in Length, and bend it double; perforate this Rectangle in the middle with the Point of a Needle, and rub the oppoſite holes ſmooth with a Whetſtone, that no roughneſs may remain about the Edges, and black them with the ſmoak of a Candle, that no brightneſs may continue within. Put the little *Sphere*, ſtill adhering to the Steel Wire, into the Holes within the Braſs Plate, and fix it there by faſtning the two Leaves of the Plate together. After this manner you may make ſeveral *Microſcopes* with great eaſe, out of which you may make choiſe of thoſe that are beſt.

The principal Uſe of this ſort of *Microſcopes*, is to look at Corpuſcles that are pellucid. And they are placed in a *Machine* made in ſuch manner, that by turning a Screw they may approach to or recede from the Object, and ſo be brought to the due diſtance, which is requiſite for diſtinct Viſion. And to this it conduces very much, that the too great Light be reſtrain'd, and only admited thro' the Hole, which is about four times the diſtance from the Object. For by

by this means the Aperture of the *Lens* is better limited, than by the breadth of the contiguous Hole, which there is no neceſſity at all for ſtraitning. The Eye muſt be brought as cloſe to the little *Sphere* as may be, that it may comprehend the greater Space.

The Corpuſcles or Drops of Liquors which are to be look'd at, are put upon a little circular plane of Glaſs, which is made to ſlide laterally every way, that we may bring every part of the Object to be view'd ſucceſſively. Some attract the Liquor to be examin'd into *capillary* Tubes of Glaſs ſo ſmall as ſcarce to admit an Hair, which has likewiſe its Uſes. But in uſing thoſe little *Lens*'s before mention'd, care muſt be taken that while by means of another *Lens* on one ſide we caſt Light upon the Object, the Hole of the apperture may be exactly limited, by trying how much it may lie open without being an Hinderance to diſtinct Viſion. For here the Points of Corpuſcles emit Rays of Light, and are ſo many *Radiant* Points, which is quite otherwiſe in thoſe pellucid Corpuſcles that are look'd at thro' little *Spheres*, where the Objects intercept the Light, not emit it.

The Effects of this ſort of little *Lens*'s and *Spheres* are very wonderful, as may be ſeen from thoſe Experiments with them which have been made publick, and from which

our Knowledge of Nature has receiv'd very great Light and Information. By ſeſe the Circulatory Motion of the Blood has been put beyond controverſy, which our *Lewenhoek*, the moſt diligent Obſerver of theſe Matters, has ſhewn me in the Tail of an Eel to my very great ſatisfaction and delight. For the Blood appears pellucid, and conſiſting of reddiſh Globules, and runs thro' the Channels of the Arteries, which are continu'd to the Veins with a very rapid Motion. Which without doubt might be obſerv'd in all other Animals, if we cou'd find out ſuch parts in them as are pervious to the Light. He put the live Eel into a Glaſs Tube half full of Water, to which he externally applied the *Microſcope* at that part where the extremity of the Tail touch'd the Tube.

'Tis alſo very pleaſant to obſerve the *Animalcula* that ſwim in drops of Water, in which we have infus'd Ginger, Pepper, or ſomething elſe of an hot powerful Odour for ſome Days. They are of various Forms, and ſome leſs than others; their Motions are wonderful and for their ſize ſufficiently quick, nor is the Inſtrument apparent by which they perform them, for they have neither Legs nor Arms, nor do they bend their Bodies like Fiſhes. For the little Eels in Vinegar, which are much larger than they, ſwim like thoſe in the River, in which

it

it is very much to be wonder'd that they ſhou'd generate little ones of themſelves. For I ſaw one which had four young Ones within it, (for they are altogether pellucid) and after it had been kept in the Tube for ſome Hours brought them all forth, every one of which did afterwards ſwim by it ſelf.

It is very probable that thoſe *Animalcula* which I have ſaid move about in Water, are invited thither out of the Air by the Odour of the Infuſion. For the ſame Figures appear upon macerating ſeveral things in Water, but if the Veſſel be cover'd none at all appear. Nor is it difficult to conceive how they ſhou'd be ſupported in the Air, when they are ſo much ſmaller than the fineſt Duſt that is. So that perhaps we draw many thouſands of them into our Lungs every time we fetch our Breath, without knowing it. Nor wou'd it be uſeleſs to obſerve at what time of the Year they appear in greateſt Numbers, and whether they encreaſe in a vitiated Air. Milk appears to conſiſt of ſmall pellucid Globules ſwimming in a Liquor likewiſe pellucid, but of a different refraction; and hence it is that it appears white, tho' it contains no other matter but what is perfectly tranſparent, and without Colour.

I omit thoſe many wonderful Forms of minute Inſects; the Wings of Butter-flies

and Gnats, cover'd with little Feathers; the Powders obſerv'd in the middle of Flower-Tops, which are nothing elſe but little tranſparent Bladders fill'd with that matter, of which the Bees make their Honey, and which they carry between their Legs into their Hives. But what ought to be look'd upon as the moſt wonderful and aſtoniſhing of all, is that an immenſe multitude of *Animalcula* are diſcover'd to ſwim in the *Semen Maſculinum*, after the manner of little Fiſhes, almoſt of the ſame Figure with a Frog newly form'd and yet without Legs. Which *Animalcula*, I make no queſtion, enter the *Ova Muliebria*, and are the Rudiments of what is brought forth from thence. There are ſeveral Conſiderations which confirm this Opinion, nor is it any great Objection that out of ſo great a Multitude either few or only one of them comes to maturity and grows to be an Animal; ſince the ſame abundance and ſuperfluous fruitfulneſs is equally obſervable in moſt Seeds of Trees and Herbs, as of Firr, Poppies, *&c.*

These *Animalcula* by reaſon of their wonderful ſmallneſs (for even ten Thouſand of them are not equal to the ſmalleſt grain of Sand) ought to be look'd at thro' ſuch Glaſs Globules as have the greateſt power of Magnifying.

PROB.

Prop. II.

To explain the Effects of Compound Microscopes.

We come now to speak of *compound Microscopes*, by the help of whieh such Objects as are not transparent are look'd at, and their true Colours discover'd, and that much better and more commodiously than thro' single *Lens*'s.

[*Fig.* 12. 13.] Suppose the *Microscope* be a double one, consisting of two *Lens*'s, one less A, and another greater B. Why we dispose them so, we shall afterwards explain. And let B be the ocular *Lens* nearest the Eye, plac'd suppose at C; A the Object *Lens* nearest the Object placed suppose at E; and A B C the common Axis of both *Lens*'s. There will be two Cases, as may be seen represented in the two Figures to which this *Prop.* refers. In the first, Rays proceeding from a single point E of the Object and falling upon the *Lens* A, are refracted by it and again united in the Point P, and there inter-

interſecting one another and proceeding towards the *Lens* B, are by it refracted and made parallel, and ſo enter the Eye at C and by that means make diſtinct Viſion. 'Tis neceſſary therefore that A E the diſtance of the Object ſhou'd be greater than A Q the *Focal* diſtance of the *Lens* A. And the *Focus* P muſt be found by *Prop. XVI*, or by making EQ, EA, EP, in a continual Proportion. But the *Lens* B is to be ſo plac'd, that its *Focus* on that ſide towards A, may fall exactly upon the Point P, in order that the Rays may be made parallel after refraction at the *Lens* B. All which is eaſily done by what has been before demonſtrated. The other Figure (13.) repreſents the ſeveral Rays D A G, F A H, E A B, proceeding from different Points of the Object. A is the middle point of the Lens, and A P, A B, A C, are made in a continual proportion in order to determine the place of the Eye C; for by this means however ſmall the Aperture of the little *Lens* A may be, the whole *Lens* B will nevertheleſs be fill'd with the Image of the Object, becauſe the Rays falling from A upon the whole *Lens* B, are collected in the Point C.

[*Fig.* 13.] But the proportion of the apparent Magnitude to the true, will be found by drawing the right Line C F. For the proportion required will be the ſame with that which the Angle B C H bears to the Angle E C F; which proportion is compounded of

the proportion of the Angle BCH to BAH, and of the Angle BAH or EAF to the Angle ECF. But the firſt of theſe is the ſame with the proportion of the right Line AB to BC, and the laſt that of CE to EA, becauſe Angles that are ſmall are look'd upon to be to one another as their Tangents. Therefore the proportion of the apparent Magnitude to the true, will be compounded of the proportions AB to BC or AP to PB (for AP, AB, AC are in a continual proportion) and CE to EA. But that the Effects of the *Microſcope* maybe more exactly eſtimated, the Angle BCH is rather to be compared with the Angle, under which the right Line EF would be ſeen at the diſtance of 8 Inches from the Eye, that is with the Angle ELF, LE being taken equal to 8 Inches, according to what has been ſaid before of magnifying by a ſingle *Lens*. And conſequently the proportion of magnifying muſt be here underſtood to be compounded of the proportion of the Angle BCH to BAH, and BAH or EAF to ELF; that is, of the proportion of AP to PB, and of EL, a line 8 Inches long, to the right Line EA. For if the *Microſcope* were of ſo great length, that for example, CE ſhould be two foot long, that is equal to thrice the right Line LE; and the apparent Magnitude to the true, were found by the former Reaſoning to be as 90 to 1, yet it is not really any greater than as 30

to

to 1; becauſe the right Line E F wou'd only appear 30 times greater by the Aſſiſtance of the *Microſcope*, than it wou'd if view'd by the naked Eye at the diſtance of 8 Inches. For we are not to conſider how much, by means of the *Microſcope*, we magnify an Object at the diſtance of two Feet; but how much greater it is made, than when view'd at that diſtance, to which we bring our Eye when we deſire to look at any thing more curiouſly.

Of

Of the Light *and* Aperture *of* Microscopes.

UPon the *Aperture* of *Microſcopes* all their Effects and Virtue entirely depend; ſo that from hence it is that we are to learn to what degree the magnifying of Objects may be brought; which no Body that I know of, has hitherto determined. And it will be found that we may here proceed *ad infinitum*, as ſhall be ſhown in *Teleſcopes*, not indeed in a ſingle *Microſcope* of one little *Lens*, but in thoſe which are made by a combination of more than one.

In *Microſcopes* made of a ſingle Lens, it is to be obſerv'd, that if their *Focal* Diſtance be about half an Inch or greater, there will be no occaſion for limiting the *Aperture* in order to make diſtinct Viſion; becauſe the very narrowneſs of the Pupil of the Eye excludes, as much as there is occaſion, thoſe Rays that diſturb Viſion, and as much as they wou'd be excluded if the *Lens* were made to have a leſs *Aperture*. But in ſmaller *Lens*'s where this Limitation of the *Aperture* is neceſſary, the Rule is that the Di-

ameters of thoſe Apertures ſhou'd be in the ſame proportion with the *Focal* Diſtances of the reſpective *Lens*'s, in order to have the Object ſeen by both equally diſtinct. But the *Light* or *Brightneſs* will be in a duplicate proportion of thoſe *Focal* Diſtances; ſo that the more convex the *Lens* is, the greater indeed, but then the more obſcurely will every thing be ſeen,

[*Fig.* 14.] Let P be a ſmall *Lens*, whoſe Axis is TBF, PD the Semidiameter of the Aperture, which Experience teaches to be the greateſt that can be admitted, and that leſs than the Pupil of the Eye, F the extreme *Focus* of *Red Ray*, (which are leaſt refrangible) proceeding parallel to the Axis, in which Point ſuppoſe the Object to be placed, and B the *Focus* of *Violet colour'd Rays, which are moſt refrangible.* The ſame things being ſuppoſed in a ſmaller *Lens p*, the Semidiameter of whoſe *Aperture pd* is to the *Focal* Diſtance *pf* in the ſame proportion as in the greater; I ſay the Object will be ſeen equally diſtinct in both.

For ſince the Ray ED parallel to the Axis falling upon the Lens P is refracted unequal and divided into its extreme Colours by the Angle FDB, ſo that the extreme Colour Red paſſes to F, and the extreme Violet-Colour to B; it will happen on the contrary, that a Ray FD proceeding from the Object will be divided into it's extreme

treme Colours by an Angle E D K, equal to F D B. Therefore in both Cases F D B is the *Angle of Aberration*, upon which depends the Aberration of the Rays in the bottom of the Eye, as shall be shewn when we come to speak of *Telescopes*. But since from the Nature of this *Aberration*, P F is to F B, as *pf* to *fb*; and also by construction P D is to P F, as *p d* to *pf*; it follows that the Angles as well P F D, *pf d*, as P B D, *p b d* are equal. Wherefore the difference of the Antecedents, P F D, P B D, is equal to the difference of the Consequents *pf d*, *p b d*; that is, the Angle F D B is equal to the Angle *f d b*, and consequently the *Aberrations* in the bottom of the Eye are in both Cases equal, and by that means Vision equally distinct.

Moreover because the Angles P F D, *pf d* are equal, it is plain that the same quantity of Rays in both cases proceeds from the same points of the Object F and f or any others, upon the *Lens*'s, and from thence to the Eye. But the Breadth of the Object in the Bottom of the Eye is in the smaller *Lens* so much greater, as P F is greater than *pf*, as has been before demonstrated; and the apparent Surfaces are in a duplicate proportion of those Breadths. Therefore the same quantity of Lucid Rays expended towards illustrating each Surface, will make that which is least

the cleareſt by ſo much as the other Surface is greater, that is, in a duplicate proportion of P F to *p f*, which was the laſt thing to be demonſtrated.

Since therefore the ſame perfection of Viſion which is to be found in larger *Lens*'s, cannot be had in more convex *Lens*'s without diminiſhing the Brightneſs of the Object at the ſame time; It follows that we cannot proceed in magnifying as much as we pleaſe, unleſs a greater Light be borrow'd ſomewhere elſe to illuſtrate the Object. Nor will this be of any great Benefit, becauſe the Latitude at the Pupil of the Eye, or the little Cylinder of Rays flowing from every point of the Object, and which has here the ſame Latitude with the Aperture, cannot be contracted farther than the fifth or ſixth part of a *Line*; ſo that even this limits the Efficacy of theſe little *Lens*'s.

The Effects of more compounded *Microſcopes* will eaſily be accounted for, after the ſame manner. And indeed a full Conſideration of *Prop*. XXI. and XXII. is ſufficient for explaining the Effects of all ſorts of compound *Microſcopes*.

Of

Of TELESCOPES.

PROP. III.

A Telescope *made by a* convex *and* concave Lens *represents vastly distant Objects distinct and erect; and magnifies them according to the Proportion of the* Focal *Distance of the* convex Lens *to the* Focal *Distance of the* concave Lens.

Fig. 15. Let A O be the common Axis of both *Lens*'s; and A the extreme *Convex Lens*, whose *Focus* of parallel Rays proceeding from the vastly distant *Object* is suppos'd to be at O. Let D be the *Concave Lens*, which is so placed between the *Lens* A and its *Focus* O, that the same Point O may also be the *Focus* of the *Concave Lens*, where Rays falling parallel from the side of O would be collected. And first suppose the Eye of the Spectator placed next to this *Lens*.

The Rays then proceeding parallel from each Point of the *vastly distant Object*, and falling upon the *Lens*, those which proceed from

from that Point of the *Object*, which is in the Axis produced, would be collected at the Point O; but they are again made parallel by means of the Lens D. We would have the Rays fall parallel upon the Eye, that the Telescope may be fitted for those who have good Eyes; For we shall speak afterwards of short-sighted Eyes. In like manner the Rays proceeding from those Points of the *vastly distant Object* which are out of the Axis, would be all collected at respective Points near O; but these also by *Refraction* at the *Lens* D are again made parallel, tho' something Oblique to the Axis A D, which Rays, to avoid Confusion, are not express'd in the Figure. Therefore the Rays which proceed from the *vastly distant Object* being made to fall parallel upon the Eye, will make *distinct Vision*; and since those Rays that proceed from the *Object* go on to meet the Eye in the same order, 'tis plain the apparent Popofition of the *Object* will be the same with the true, or the *Object* will be *erect*.

Fig. 16. The *Lens*'s A C and D, and the Point O being placed as before, find by *Prop. XVI.* the Point P, to which Rays tending, will by *Refraction* at the *Lens* A C be collected at D the Centre of the *Lens* D: Which Point is also found, by making DP a third proportional to D O, DA, and taking it on the same side with

DO.

DO. Suppoſe the Ray ECP to be one of thoſe which proceed from the extreme right ſide of the *vaſtly diſtant Object*, which imagine to be the Moon, and its Centre to be placed in the Axis D A produced. It is plain that this Ray will come to the Eye in the Right Line CDF, becauſe it paſſes thro' the Centre of the *Lens* D, whoſe middle Thickneſs may be neglected as inconſiderable, and its two Surfaces in that place look'd upon as parallel. But we have ſhewn before that all the Rays proceeding from each Point of the Moon, will by means of ſuch a Teleſcope fall parallel upon the Eye. Wherefore the Eye will receive all the Rays from that Point E of the Moon in ſuch manner, as that they ſhall be parallel to the Ray CDF; and conſequently will ſee that Point of the Moon in the place to which the Right Line DC tends, which tending to the ſame ſide of the Axis, on which that point of the Moon is ſituated, from whence the Rays proceeded, 'tis plain the Object will appear *Erect*. Moreover the Angle ADC determines the Semidiameter of the Moon, as encreas'd by the Teleſcope. But the Angle CPA is that which determines its Semidiameter, as ſeen by the naked Eye, becauſe we before ſuppoſed the Ray ECP to proceed from the extreme right ſide of the Moon, and the Ray HAP from its Centre: For tho' the Point P is beyond the Eye,

Eye, and the Eye sees from the Point O, yet the Moon being an Object vastly distant will appear under the same Angle to the naked Eye, whether it be view'd from the Point P or O. Therefore the Moon will appear magnified, according to the Proportion of the Angle ADC to APC, which Proportion may here be look'd upon as the same with that of PA to DA. But because by Construction, DO is to DA as DA to DP; by inverting and compounding the Proportion AO will be to OD, as PA to AD. Wherefore the apparent Magnitude will be to the true as AO to OD. *Q. E. D.*

It appears from hence that the apparent Magnitude is the same, in whatsoever place behind the *Lens* D the Eye is situated.

Fig. 17. Let the *Lens*'s AC and D be placed as before, and let AQ be taken in their Axis produced equal to AO. And out of those Rays which proceed from a Point of the right side of the Moon, let us consider the Ray RQC passing thro' the Point Q, (for some one will pass thro' it) and meeting with the *Lens* AC in C. It will afterwards become parallel to the Axis AD, and when refracted again at the concave *Lens*, will diverge, as if it came from the Point L, and will tend to the Eye in the Right Line LIF, so as that the Distance LD may be equal to DO, be-

cause

cause L is in that Case the *Focus* of parallel Rays falling upon the *Lens* D.

The Proportion of the encreas'd Magnitude is hence easily collected. For because the Rays proceeding from the right extreme of the Moon, after having passed both *Lens*'s, come parallel to the Pupil G F, and consequently they become parallel to the Ray L I F, which we know to be one of them; that Point of the Moon will appear in the Right Line I L, and consequently the Semidiameter of the Moon will be comprehended in the Angle I L D. But the Angle in which the Semidiameter wou'd appear to the naked Eye, either from D or from Q, is R Q H, or C Q A. Therefore the Proportion of the encreas'd Magnitude is the same with that of the Angle D L I to A Q C; that is, because of A C, D I equal, as A Q to L D. But A Q is equal to A O, and L D is equal to D O. Therefore the Proportion of the apparent Magnitude to the true is as A O to O D. *Q. E. D.*

To determine what will be the Amplitude of the visual Angle, *or of the Space which is represented at one view by a* Telescope *consisting of a* Convex *and* Concave Lens.

Fig. 18. The Amplitude of the *visual Angle* in these *Telescopes*, depends chiefly upon the Magnitude of the Pupil of the Eye, which is confirm'd by Experiment. For if applying your Eye to the Telescope, you first shut it that the Pupil may be dilated as it usually is in the Dark, and then open it on a sudden; at first view you will discern Objects in a larger Orb than a little while afterwards, the Orb being presently contracted as soon as the Pupil is contracted by the Brightness of the Light. But if you place a Plate perforated with a small Hole before the Eye, you will discern every Object in a lesser Orb.

If you make the Hole extremely small, the lucid Orb will not be contracted in proportion to the smallness of the Hole; but its Amplitude will then be limited by the Aperture of the Convex *Lens*, and consequently will not be diminished beyond a certain Degree, except the Convex *Lens* be also more contracted. The Reason of which is very easy to be explain'd. For if E F be the convex *Lens*, and B the concave, to which the Pupil of the Eye apply'd has first the Magnitude C D: Draw from the opposite Points C, D in the Circumference of the Pupil, through the Centre of the *Lens* A the right Lines C A H, D A G. These will determine the *visual Angle*, under

der which that part of any Object which is ſeen at one view is comprehended: Becauſe Rays coming from the Points G, H thro' the Centre of the *Lens* A, penetrate without Inflection to C and D; therefore that part of the Object which is comprehended within the Angle G A H, cannot but ſend Rays to the Eye, even tho' the Pupil were a little narrower than D B C. For drawing G A K ſo as to make A K equal to A O, and joining E K; If ſo be E K falls upon the Pupil, the Object comprehended under the Angle G A H will be diſcern'd, but the extreme Points towards which the right Lines A G, A H tend, will be ſeen but obſcurely, becauſe only a ſmall part of the Rays, which they caſt upon the *Lens* E F enter the Pupil. And hence it happens, that how much ſoever the Aperture of the *Lens* E F is contracted, the Amplitude of the *viſual Angle* is nevertheleſs not at all, or extremely little diminiſhed, ſo the Orb of the Pupil be not contracted. But this breadth of the Pupil being diminiſhed, and reduced as it were to a Point, the Amplitude of the *viſual Angle* is the ſame with that of the Angle E P F; E F being ſuppoſed the Aperture of the convex *Lens*, and the Point P found by *Prop. XVI.* or by making B O (the Diſtance of the concave *Lens* from the *Focus*

of the convex) BA and BP in a continual Proportion. For no Rays, transmitted through the *Lens* A can arrive at the Point of the Eye B, but such as before they fall upon that *Lens* tend towards the Point P. The greatest Angle EPF of which Rays is determin'd by the Aperture of the *Lens* A.

This is the *Telescope* which was first found out by *Galilæus*, and still retains his Name: And is the same with a common Perspective Glass.

PROP.

Prop. IV.

A Telescope made of two Convex Lens's *represents vastly distant Objects distinct but inverted, and magnifies them according to the Proportion of the* Focal *Distance of the Exteriour or* Object *Lens, to the* Focal *Distance of the Interiour or* Ocular *Lens.*

[*Fig.* 19, 20.] Let AC be the exteriour convex *Lens*, D the interiour, AD the common Axis of both, and O the *Focus* of the *Lens* AC. Let the other Convex D be so placed, that the same Point O may be also its *Focus*, or the Point of concourse of parallel Rays coming from the side of G, where the Eye is supposed. We are to shew that all this being supposed, vastly distant Objects will be seen distinct, and inverted and magnified, according to the Proportion of AO to OD.

And

And here we muſt make uſe of two ſeveral Figures, as in the *preceding Propoſition*, in the firſt of which the Rays coming parallel to the Axis H A, are by the Refraction of the Lens A C collected at its *Focus* O, and from thence tending farther to the *Lens* D, are by it again made parallell to the Axis A D, and ſo come to the Eye placed at G. And as in the *preceding Prop.* we muſt again conſider this compoſition of parallel Rays, as coming from a ſingle Point of the vaſtly diſtant Object, which is placed in the Axis H A D, as ſuppoſe from the Centre of the Moon; and the like parallel Rays coming from every other Point of the *Object* upon the *Lens* A C, as ſuppoſe from the extreme right ſide of the Moon, which are inclin'd to the former, and being thereby refracted, are collected in a Point of the Axis near O, where interſecting themſelves, and proceeding to the *Lens* D, they are again made parallel, (that is, only among themſelves reſpectively) and ſo arrive at the Eye. Whence 'tis plain, Viſion will be made diſtinct.

The other Figure ſhews the inverted Situation, and the Proportion of the encreas'd Magnitude of the *Object*. Where the convex *Lens*, A C and D, and their common *Focus* O being placed as before; and moreover, as in the ſecond Demonſtration of *preceding Prop.* the Diſtance A Q being made

made equal to A O; the remaining part of the Demonstration will proceed much after the same manner. For if out of the Rays which proceed from a Point in the extreme right side of the Moon, we choose one R Q C passing thro' the Point Q: That, after refraction at the *Lens* A C will pass in C I parallel to A D, and being again refracted by the *Lens* D, will tend along the right Line I F L to the Point L, taken in such manner that the Distance D L is equal to D O. But because the Rays from the extreme right side of the Moon, after refraction at both *Lens*'s, arrive parallel at the Eye, as has been said before, and I F L is one of them; It follows that they will all fall parallel to I F L upon the Eye, and that Point of the Moon will be seen in a Place, according to the right Line F I; which since it tends to the opposite side to that from whence those Rays came, 'tis plain that the situation of the Moon will appear inverted, so that the right side will be changed to the left, and the upper Parts to the lower. Moreover since the Centre of the Moon will be seen in the right Line D A, I L D will be the apparent Angle of the Semidiameter of the Moon. But to the naked Eye, that Semidiameter is comprehended under the Angle H Q R, or A Q C. Therefore the *Ratio* of the apparent Magnitude to the true, is as the Angle D L I to A Q C, that is as A Q to D L; because A C,

DI

DI are equal, that is as AO to OD. *Q. E. D.*

And here likewise it appears that it signifies nothing to the apparent Magnitude, wheresoever the Eye is placed behind the *Lens* D. But that it may comprehend most at one view, it is convenient it should be placed at or near the Point L: Because it appears, that altho' the Breadth of the Pupil be ever so little, yet the whole *Lens* D while it does not exceed the Aperture of the *Lens* AC (for it is usually confined within this Measure) will be seen full of the Object.

This is the *Telescope* most commonly used, to look at Celestial Bodies.

PROP.

Prop. V.

TO explain the Construction of a Telescope *compounded of* four Convexes, *by means of which Objects are seen* erect *and very* amply.

[*Fig.* 21, 22.] *Telescopes* made of *two Convexes*, because of their Inverting the position of the Object, are seldom used, except in observing the Stars, the position of which is not regarded. The proportion in which this Sort magnifyes the Object has already been demonstrated. But if we wou'd have these Images again made *erect*, and at the same time a great share of them be represented to the Eye at one View very *amply*, we must use 3, 4, 5 or more *Lens*'s. Which however are not to be multiply'd without Cause, because the Matter of each of them and the Reflexion of their several Surfaces divert part of the Rays. But we cannot obtain the desir'd Effect perfectly, with fewer than 4 *Lens*'s. For altho' in the same Length of the *Telescope* both an *erect* Situation and the same degree of magnifying, and an equal share of the Object may be had as well with 3 as 4 *Lens*'s; yet the

composition of 3 *Lens*'s is much more convenient than that of 4; because in that, the two Ocular *Lens*'s, or at least that which is next the Eye must be made of larger Segments of a Sphere, with respect to its Diameter, or to the *Focal* distance, if the same Magnitude of the visual Angle be requir'd. And hence the Objects come to be colour'd, and right Lines, at the Margins of the Aperture, appear Curve. Therefore we must make our Telescope of 4 *Lens*'s, which is done after the following Manner.

The exterior or Object *Lens* is A, whose *Focal* Distance is A B, and in the same Axis are placed three Ocular *Lens*'s, C, D and E, all equal to one another, the inmost of which is placed beyond the *Focus* B, by its *Focal* distance B C, and the next D is placed beyond C, by twice that distance B C, and the last as far from D as that was from C, and lastly the Eye must be placed beyond this last by the distance B C.

There is here again occasion for two Figures, in the first of which are represented Rays proceeding from a single Point of the vastly distant Object: Which, 'tis plain to any who understand what has gone before, first fall as it were parallel upon the *Lens* A, and are by it collected at its *Focus* B, and thence diverging fall upon the *Lens* C, which makes them again parallel and throws them upon the *Lens* D, which collects them at its

Focus

Focus H, the middle Point of the distance DE, from whence proceeding on to the *Lens* E they are by it made a third time parallel, and being receiv'd so by the Eye F, they make distinct Vision by being collected at its Focus which is in the bottom of the Eye.

The other Figure considers the *Proportion* of *Magnifying*, which is, *That which AB the* Focal *Distance of the Object* Lens *bears to BC the Focal Distance of one of the Ocular* Lens's. And demonstrates likewise the Amplitude of the Visual Angle. For the Apertures of the three Ocular *Lens*'s being supposed equal, which must not exceed the Aperture of the Object *Lens* A; draw MQ, NR parallel to the common Axis, and comprehending the Diameters of the Apertures of the *Lens*'s E and D. And also KO, LP parallel to the same Axis, and comprehending KL the Aperture of the *Lens* C: and taking AG equal to AB, draw the right Lines, OGU, PGT intersecting one another in G. Now it is evident the Latitude of the Object which if seen by the naked Eye from the Point G, and consequently from F also, the distance of the Object being as it were infinite, wou'd appear comprehended in the Angle TGV; if seen thro' the Telescope wou'd appear comprehended in the Angle MFN: And consequently the proportion of the apparent Magnitude to the true, is as the Angle MFN to the Angle

TGV or PGO; that is, PO and MN being equal, as the diſtance AG to the diſtance EF; that is, as AB the *Focal* diſtance of the Object *Lens* to BC the Focal diſtance of one of the Ocular *Lens*'s. Q. E. D.

It appears moreover that the Viſual Angle MFN comprehends the ſame Latitude of the Object, with a Teleſcope made of two *Lens*'s only A and C; for that ſhare of the Object which is comprehended in the Angle TGV, wou'd be ſeen thro' that Teleſcope in the Angle KSL equal to the Angle MFN.

This incomparible Compoſition of *Lens*'s was found out by I know not whom at *Rome*, and may be much improv'd by placing an *Annulus* or Ring either at H, the common *Focus* of the *Lens*'s D and E, or at B the common *Focus* of the *Lens*'s A and C; which is eſpecially of very great Uſe in meaſuring the Diameters of Planets. For this *Annulus* does therefore exactly circumſcribe the Circle of the apparent Images, becauſe it cuts off thoſe irregular Rays which are not collected near enough to B or H, and conſequently are not by means of the ſucceeding *Lens*'s ſent parallel to the Eye, which diſtinct Viſion requires; And the Colours likewiſe near the Margins are by this Contrivance taken away, which without it are not well to be avoided.

It

It may seem a little strange that the Colours of the Iris arise no more in this Telescope by the Refraction of so many Ocular Lens's, than in that where there is but one; but to any one that will consider it, the Reason will be very Obvious. For the *Lens* QR corrects and takes away those Colours, which the *Lens* KL produc'd, their Spherical Surfaces being equal by Construction.

Of the APERTURE *of the* LENS's.

Since the proportion of magnifying in *Telescopes* made of two *Lens*'s has been shewn to be that which the *Focal* Distance of the Object *Lens* bears to the *Focal* Distance of the Ocular *Lens*, it may be thought perhaps that however short the *Telescope* be, the Object may be magnify'd in any assigned proportion. But there are two Causes which make this impossible; One is, that the *Aperture* of the Object *Lens* remaining the same, the more we magnify the Object by using a less Convex Ocular *Lens*, the more obscure we make them appear. The other is, that it represents them less distinct. And if we expect a Remedy by encreasing the *Aperture*, the Confusion will be the more encreased. What belongs to the *Brightness* or *Obscurity*, will be easily understood by attentively considering the Image of any Object painted upon the Bottom of the Eye; which the

greater

greater it is made, whether by means of the Refraction of *Lens*'s, or only by approaching nearer, in so much greater plenty must the Rays from every Point be receiv'd within the Eye, in order that the same *Brightness* may still remain. For if looking at an Object with the naked Eye, you approach to it twice as near, the Image at the Bottom of the Eye will be twice greater in Diameter, and four times in Area. But four times more Rays do also from every Point of it enter the Pupil of the Eye; because the Angle made by the Cone of Rays becomes twice as large; and therefore it is that the same *Brightness* of the Image is perceiv'd at both Distances, which is the Contrivance of Nature. But if a *Telescope* were to be made which shou'd magnify the Diameter of any Object ten times, and represent it as bright as when it is look'd at with the naked Eye; the Diameter of the Aperture of the Object Lens ought to be ten times greater than the Diameter of the Pupil, altho' no part of the Rays were intercepted by the Reflection of the Surfaces of each *Lens*, or by the Colour of the Glass. For by this means, when the Surface of the Object is magnified an hundred times we have also an hundred times more Light than was receiv'd by the naked Pupil.

But a much less Measure of *Brightness* suffices for *Telescopes*; for those which we use in the Day-time, are not too obscure, if

they

they have but $\frac{1}{6}$ or $\frac{1}{7}$ of that *Brightneſs* which is uſuallya perceiv'd by the naked Eye. But thoſe longer ones with which we obſerve the Moon and the Planets, require not above half this laſt *Brightneſs*, becauſe the Eye is moved with a much leſs Brightneſs in the Night than in the Day. So that in a *Teleſcope* 30 Feet long, which magnifies the Diameters of the Planets 109 times, and wou'd conſequently require the Diameter of the Aperture of the Object *Lens* 109 times greater than the Diameter of the Pupil, that is of about 11 Inches, if we ſuppoſe the Diameter of the Pupil to be $\frac{1}{10}$ of an Inch; we find that an Aperture of 3 Inches in Diameter ſuffices, which admits leſs than $\frac{1}{13}$ of that *Brightneſs* which wou'd be admitted by an Aperture of 11 Inches.

The Proportions between the *Focal* Diſtances of the Object *Lens*, (which is likewiſe the Length of the Teleſcope) the *Aperture* of the ſame Object *Lens*, the *Focal* diſtance of the Ocular *Lens*, and the apparent Magnified Diameter of the Object, for *Teleſcopes* from the Length of 1 Rhinland Foot to 100, are expreſs'd in the Table following.

TABLE

The Focal Distance of the Object *Lens*, or the Length of the *Telescope*.	The Diameter of the Aperture of the Object *Lens*.	The Focal Distance of the Ocular *Lens*.	The Proportion of Magnifying, consider'd as to Diameter.
Rhinland Feet.	*Inches, & Decimals.*	*Inches, & Decimals.*	
1.	0,55.	0,61.	20.
2.	0,77.	0,85.	28.
3.	0,95.	1,05.	34.
4.	1,09.	1,20.	40.
5.	1,23.	1,35.	44.
6.	1,34.	1,47.	49.
7.	1,45.	1,60.	53.
8.	1,55.	1,71.	56.
9.	1,64.	1,80.	60.
10.	1,73.	1,90.	63.
13.	1,97.	2,17.	72.
15.	2,12.	2,33.	77.
20.	2,45.	2,70.	89.
25.	2,74.	3,01.	100.
30.	3,00.	3,30.	109.
35.	3,24.	3,56.	118.
40.	3,46.	3,81.	126.
45.	3,67.	4,04.	133.
50.	3,87.	4,26.	141.
55.	4,06.	4,47.	148.
60.	4,24.	4,66.	154.
65.	4,42.	4,86.	161.
70.	4,58.	5,04.	166.
75.	4,74.	5,21.	172.
80.	4,90.	5,39.	178.
85.	5,05.	5,56.	183.
90.	5,20.	5,72.	189.
95.	5,34.	5,87.	194.
100.	5,48.	6,03.	199.

Prop. VI.

T*O explain the Manner of fitting a* Telescope *for observing* Eclipses *of the* Sun, *and discovering the Spots in its Surface, and to determine how great its Image will be represented.*

[*Fig.* 23, 25.] A *Telescope* is found to be of great Use in observing *Eclipses* of the Sun, and also in discovering the Spots which are said to be in its Surface; by receiving the Image form'd by both it's *Lens*'s upon a white Plane, from which the Light is every other way excluded. In order to explain which Invention we must first demonstrate the Position of the *Lens*'s which is necessary to form the Image of the Sun, as clear and distinct as may be.

Let A B be the convex *Lens* next the Sun, whose *Focus* is E. The other is D, either concave or convex, for either of these sorts of *Telescopes* will do the Business, tho' a Telescope of two *Convexes* is the most convenient, because we make it represent the

Images erect, while by the other sort we invert them. Let the point K be the *Focus* of the *Lens* D, where Rays coming from the side of H parallel are after Refraction by it collected, and in H suppose the white Plane placed in order to receive the Image of the Sun. Which that it may appear distinct and nicely terminated, 'tis necessary that the Rays which proceed from any one Point of the Sun, and which fall parallel upon the *Lens* A B, shou'd again be collected in one point upon the Plane. Wherefore the Distance between the *Lens*'s A B and D ought to be something greater than in the common Disposition of the *Telescope*, or than when it is fitted for a good Sight; and the position of the *Lens* B ought to be such that the Rays which wou'd otherwise tend to the *Focus* E of the *Lens* A B, may be diverted and brought to H, which may be done by *Prop. XVI.* or by taking E K, E D, E H in a continual Proportion. But in the common Disposition of the Telescope, the *Focus* K is required to coincide with the *Focus* E, as has been shewn above. So that here the distance of the *Lens's* is increas'd by the Space E K, which will always be so much less, as the Distance E H is increas'd. For the Distance D K which is the given *Focal* Distance of the *Lens* D, is divided in such manner in E, that H E is to E D, as the same E D to E K.

[*Fig.*

[*Fig*.24, 26.] How great the Diameter of the Image of the Sun will appear upon the Plane H may thus be determin'd. Draw from the Centre of the *Lens* A B to the *Lens* D the right Lines BP, BQ, comprehending an Angle equal to that, under which the Sun's Diameter appears without a *Teleſcope*; And make BG a third Proportional to BK, BD, and join GP, GQ, which will conſequently give G the *Focus* of Rays diverging from B after refraction at the *Lens* D, which produce till they meet the Plane placed at H in the Points L, M. I ſay LM will be the Diameter of the Sun's Image repreſented upon the Plane LHM. For produce P B, Q B towards O and N. Therefore ſince from the extreme Point of the real Diameter of the Sun on the right Hand ſide, Rays are ſent upon the whole *Lens* AB, which are all to be look'd upon as parallel among themſelves, and to the right Line O B; one of them will proceed along the right Line OB, and penetrating the *Lens*, go on in the right Line BP, becauſe B is the Centre of the *Lens*, whoſe Thickneſs is here neglected. For the ſame Reaſon, one of the parallel Rays from the left extreme of the Sun's Diameter, will proceed along the right Line NBQ. But moreover, both will be refracted in ſuch manner by the *Lens* D, that diverging from

their *Focus* G, they will proceed along the right Lines PL, QM, which are the right Lines GP, GQ produced. Therefore 'tis plain the Point in the right Hand extreme of the Sun will be represented at L, and the opposite Point in the left extreme at M. For since the Image of the Sun is required distinct, it is necessary, that where one Ray proceeding from any Point of it falls upon the Plane, all the rest which proceed from the same Point should be collected there also. Therefore the Diameter of the Image is LM; And by a *Telescope* made of a Convex and Concave, the Image is inverted, and by one made of two Convexes it is represented erect. *Q. E. D.*

But it must be observ'd, that the greater LM the Image of the Sun is, the *Lens*'s AB and D remaining the same, the less clear and distinct will it be. For if all the Rays descending from the Sun upon the *Lens* AB, should possess a Space in the place LHM equal to the breadth of the *Lens* AB; that is, if they were to form the Image of the Sun equal to the Aperture of the *Lens* AB, this Image would be as clear as if the Plane were enlightned by the Sun without the Interposition of *Lens*'s. No respect being had to those Rays which the *Lens*'s reflect, or by reason of their imperfect Transparency do not transmit, which perhaps

occa-

occaſions a loſs of above half the whole Number of Rays. But if the Sun's Image be made larger, which is neceſſary to be done in Obſervations of this Nature, it will then be ſo much the more obſcure. But Experience is the beſt Judge to determine in what Magnitude it will be moſt convenient to repreſent the Sun's Image in theſe Obſervations; by trying firſt one, and then another Diſtance of the Plane from the Teleſcope. Where it is to be obſerv'd, that as we encreaſe this Diſtance, the Diſtance between the *Lens*'s AB and D ought to be a little diminiſh'd, in order to preſerve the diſtinctneſs of the Image; the Reaſon of which has already been given.

FINIS.